

Introduction to Graph Deep Learning

Minji Yoon

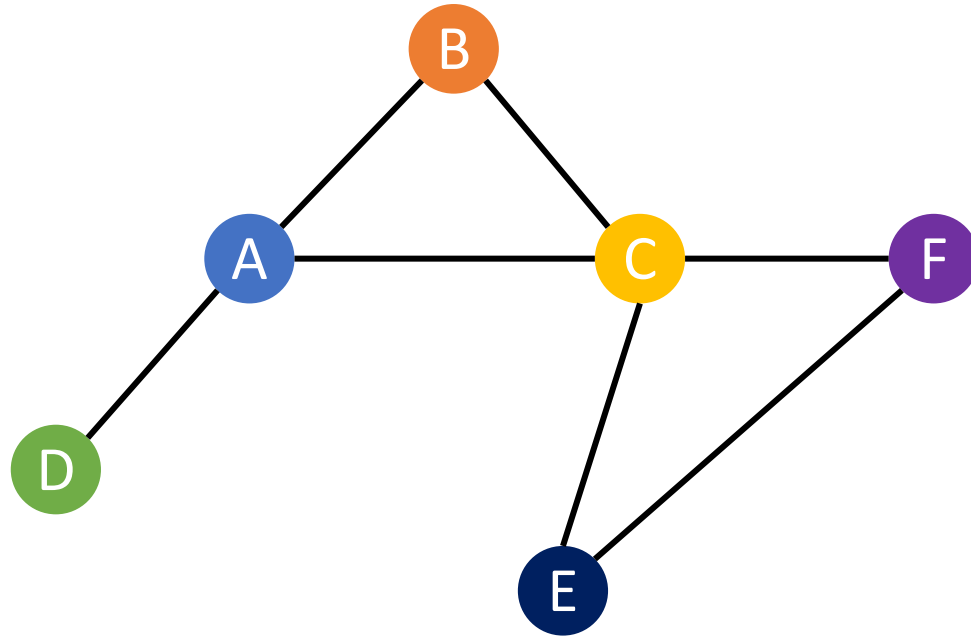
Computer Science Department

Carnegie Mellon University

Talk objectives

- Introduce Graph Neural Networks (GNNs)
- Highlight interesting open research questions

What is a graph?

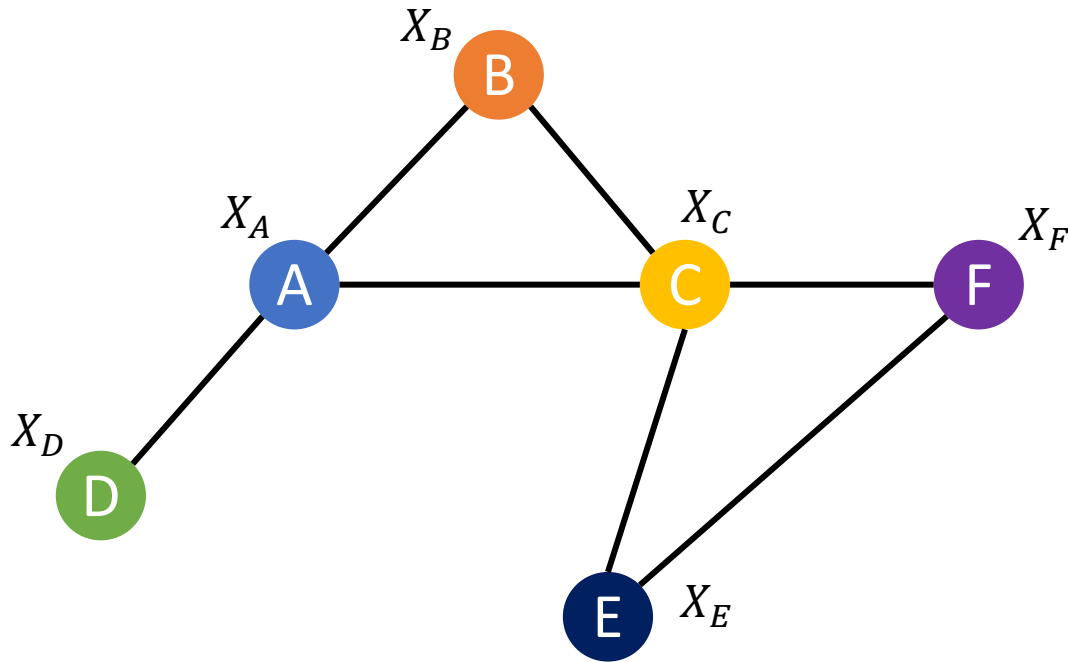


A graph is composed of

- **Nodes** (also called vertices)
- **Edges** connecting a pair of nodes presented in an **adjacency matrix**

	A	B	C	D	E	F
A		1	1	1		
B	1		1			
C	1	1			1	1
D	1					
E			1			1
F			1		1	

What is a graph?



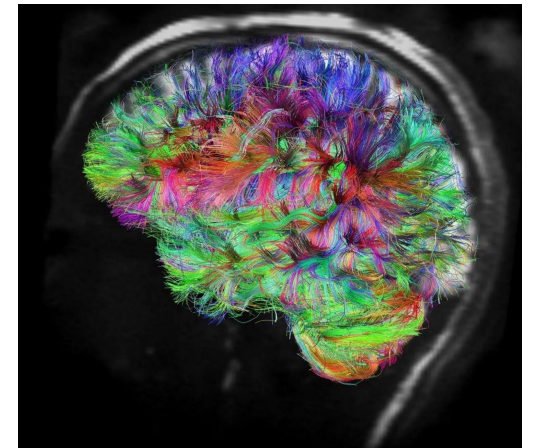
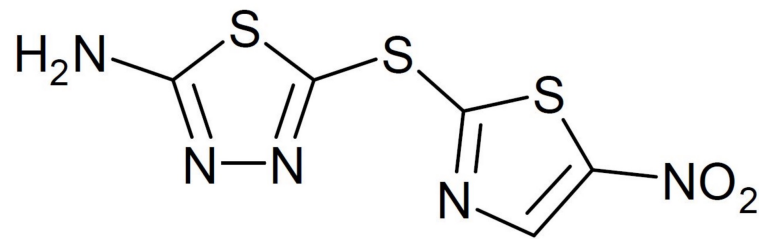
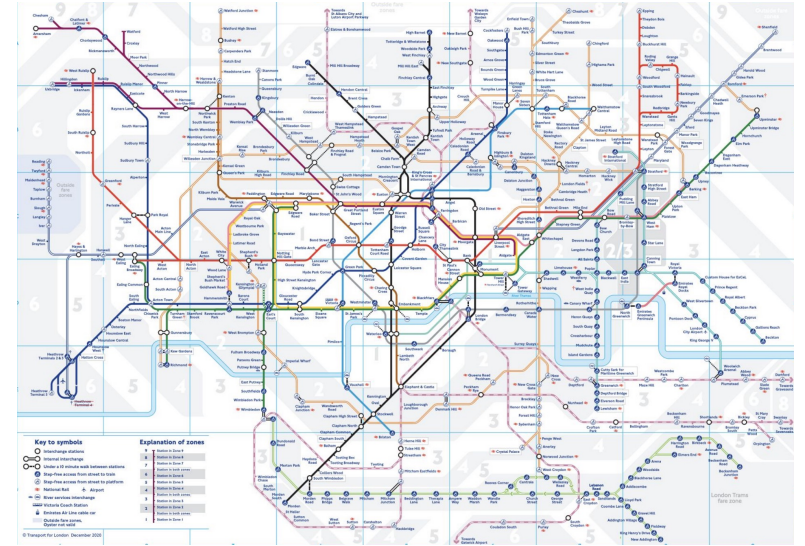
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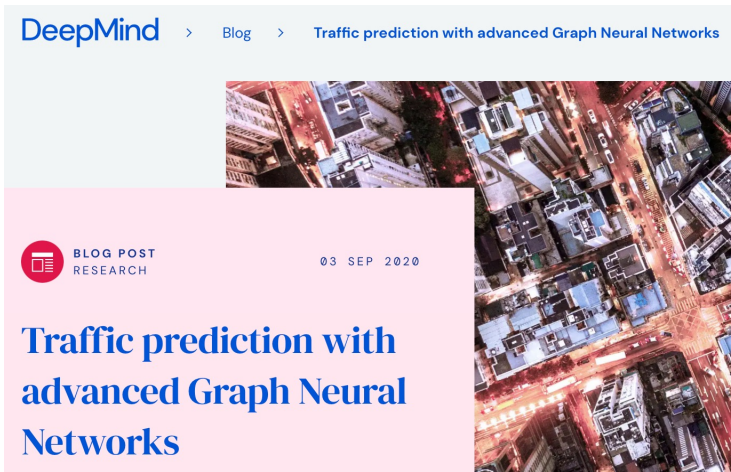
Nodes can have **feature vectors**

A	X_A
B	X_B
C	X_C
D	X_D
E	X_E
F	X_F

Graphs are everywhere



Graph Neural Networks have a large impact on...




Food Discovery with Uber Eats: Using Graph Learning to Power Recommendations

Ankit Jain, Isaac Liu, Ankur Sarda, and Piero Molino

December 4, 2019



 Pinterest Engineering
Aug 15, 2018 · 8 min read

PinSage: A new graph convolutional neural network for web-scale recommender systems

Ruining He | Pinterest engineer, Pinterest Labs

Web image search gets better with graph neural networks

Web image search uses images returned by traditional search engines in a graph neural network through which similarity signals are relieving improved ranking in cross-modal retrieval.

 | science

PUBLICATION

P-Companion: A principled framework for diversified complementary product recommendation

By Junheng Hao, [Tong Zhao](#), [Jin Li](#), [Xin Luna Dong](#), [Christos Faloutsos](#), [Yizhou Sun](#), [Wei Wang](#)
2020

Minji Yoon (CMU) - Guest lecture at 10707: Introduction to Deep Learning

Graph Neural Networks have a large impact on...

GCN-RL Circuit Designer: Transferable Transistor Sizing with Graph Neural Networks and Reinforcement Learning

Hanrui Wang¹, Kuan Wang¹, Jiacheng Yang¹, Linxiao Shen², Nan Sun², Hae-Seung Lee¹, Song Han¹

¹Massachusetts Institute of Technology

²UT Austin



The next big thing: the use of graph neural networks to discover particles

September 24, 2020 | Zack Savitsky



Machine learning algorithms can beat the world's hardest video games in minutes and solve complex equations faster than the collective efforts of generations of physicists. But the conventional algorithms still struggle to pick out stop signs on a busy street.

Object identification continues to hamper the field of machine learning — especially when the pictures are multidimensional and complicated, like the ones particle detectors take of collisions in high-energy physics experiments. However, a new class of neural networks is helping these models boost their pattern recognition abilities, and the technology may soon be implemented in particle physics experiments to optimize data analysis.

npj | computational materials

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Article | [Open Access](#) | [Published: 03 June 2021](#)

Benchmarking graph neural networks for materials chemistry

[Victor Fung](#) , [Jiaxin Zhang](#), [Eric Juarez](#) & [Bobby G. Sumpter](#)

[npj Computational Materials](#) 7, Article number: 84 (2021) | [Cite this article](#)

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A graph placement methodology for fast chip design

[Azalia Mirhoseini](#) , [Anna Goldie](#) , [Mustafa Yazgan](#), [Joe Wenjie Jiang](#), [Ebrahim Songhori](#), [Shen Wang](#), [Young-Joon Lee](#), [Eric Johnson](#), [Omkar Pathak](#), [Azade Nazi](#), [Jiwoo Pak](#), [Andy Tong](#), [Kavya Srinivasa](#), [William Hang](#), [Emre Tuncer](#), [Quoc V. Le](#), [James Laudon](#), [Richard Ho](#), [Roger Carpenter](#) & [Jeff Dean](#)

Graph Neural Networks have a large impact on...

nature

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NEWS | 01 December 2021

DeepMind's AI helps untangle the mathematics of knots

The machine-learning techniques could sets.

Patterns

Opinion

Neural algorithmic reasoning

Petar Veličković^{1,*} and Charles Blundell¹

¹DeepMind, London, Greater London, UK

*Correspondence: petarv@google.com

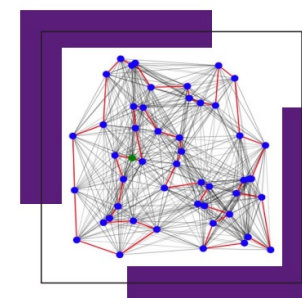
<https://doi.org/10.1016/j.patter.2021.100273>

We present neural algorithmic reasoning—the art of building neural networks that are able to execute algorithmic computation—and provide our opinion on its transformative potential for running classical algorithms on inputs previously considered inaccessible to them.

 institute for pure & applied mathematics

Deep Learning and Combinatorial Optimization

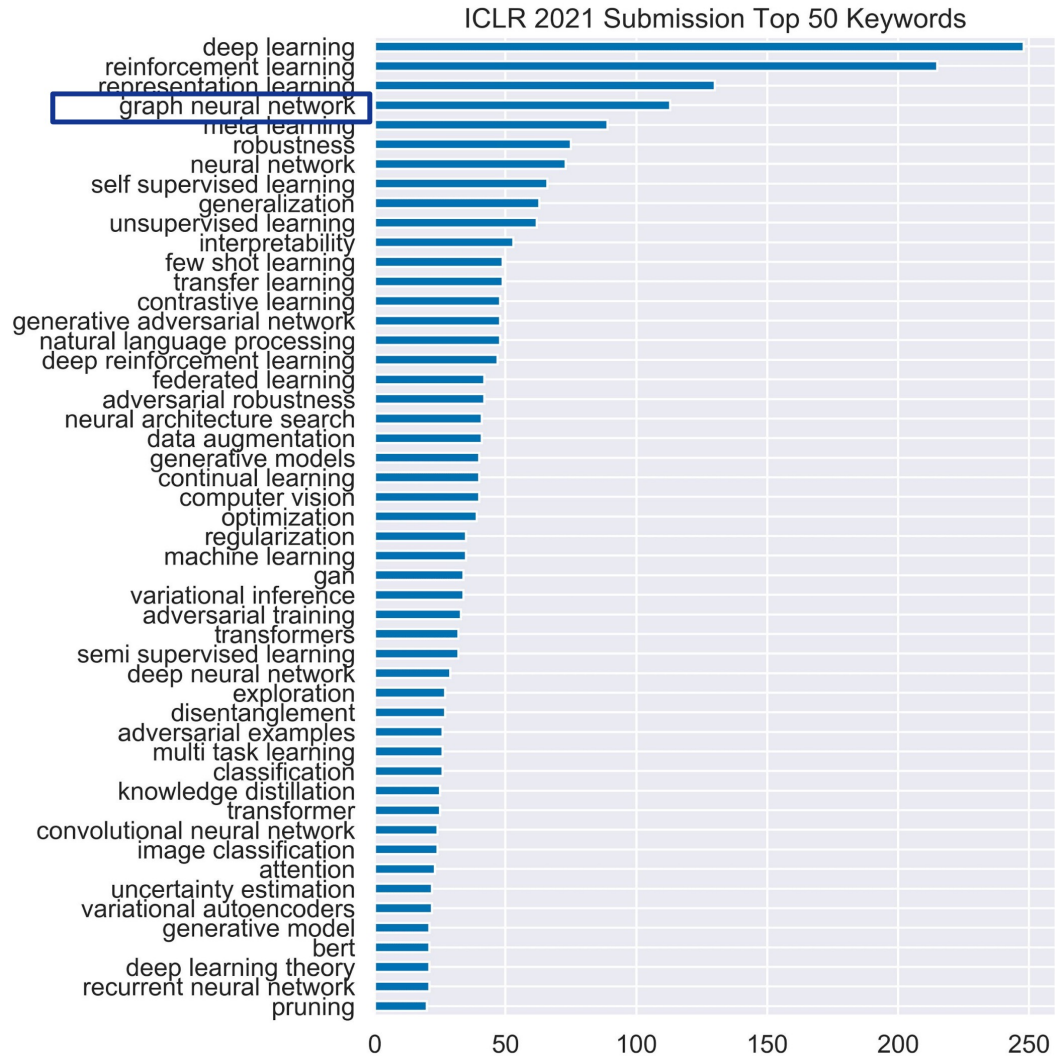
February 22 - 25, 2021



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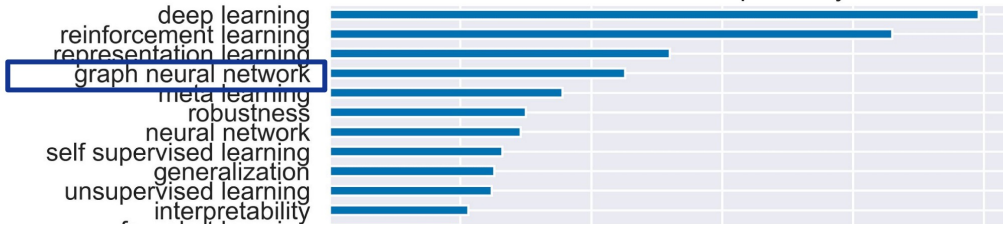


Popular research topic

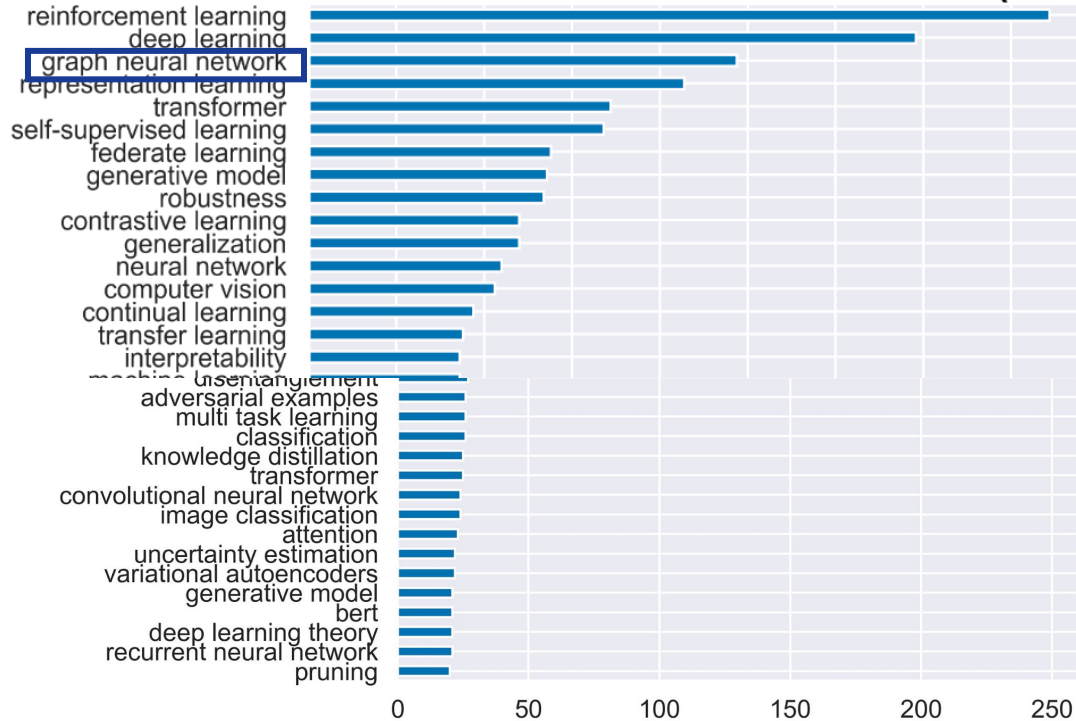


Popular research topic

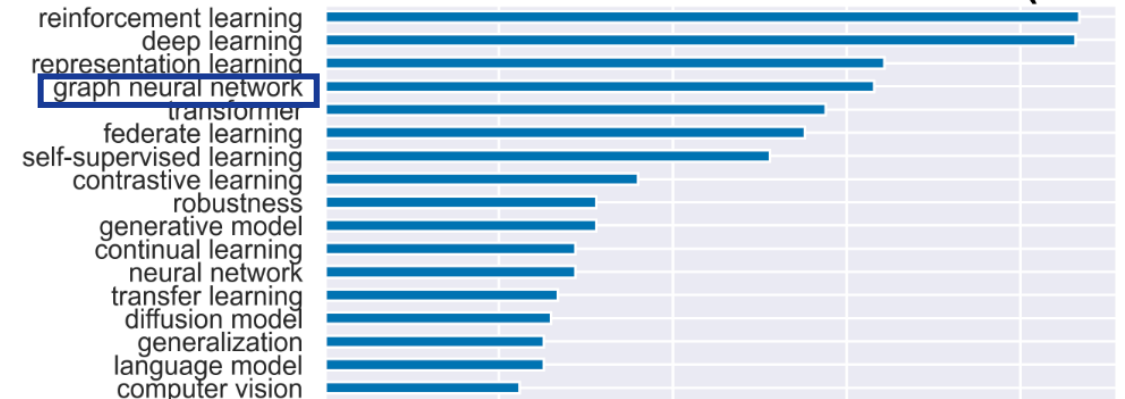
ICLR 2021 Submission Top 50 Keywords



50 MOST APPEARED KEYWORDS (2022)

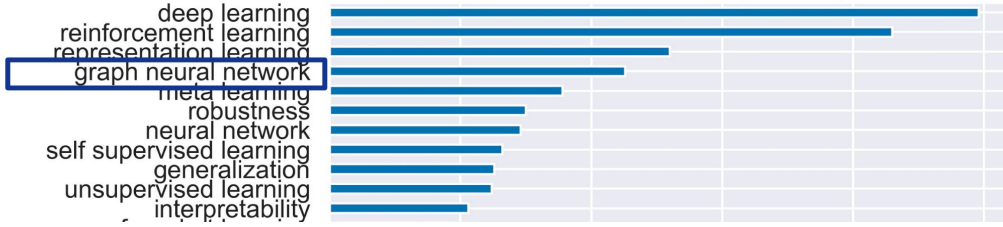


50 MOST APPEARED KEYWORDS (2023)

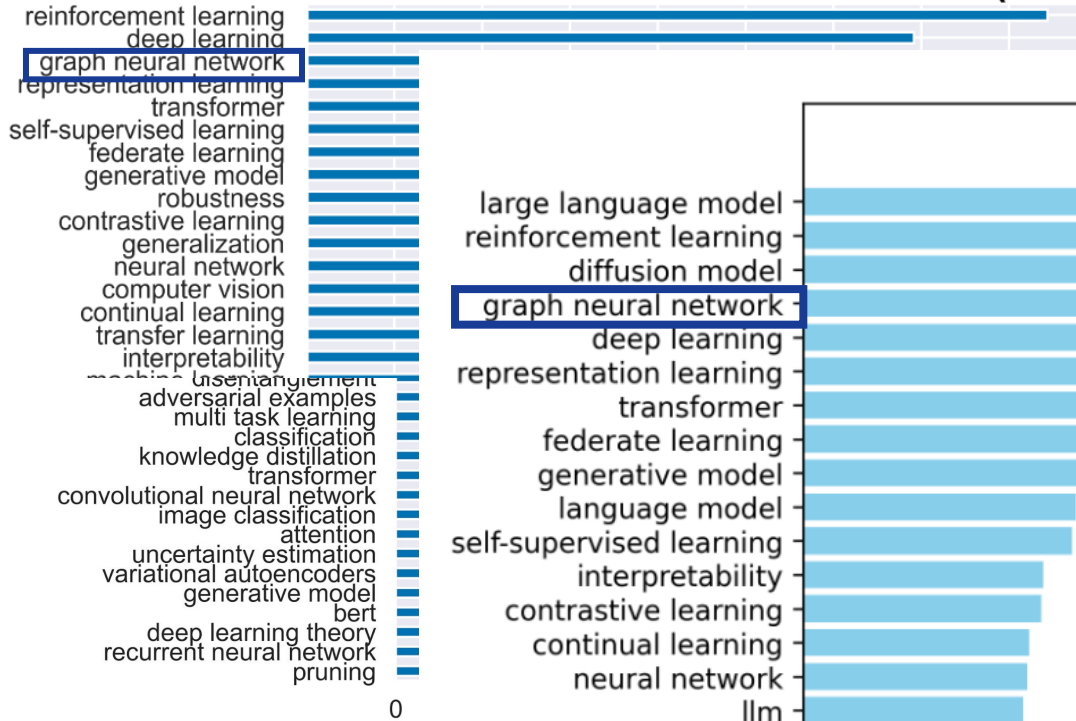


Popular research topic

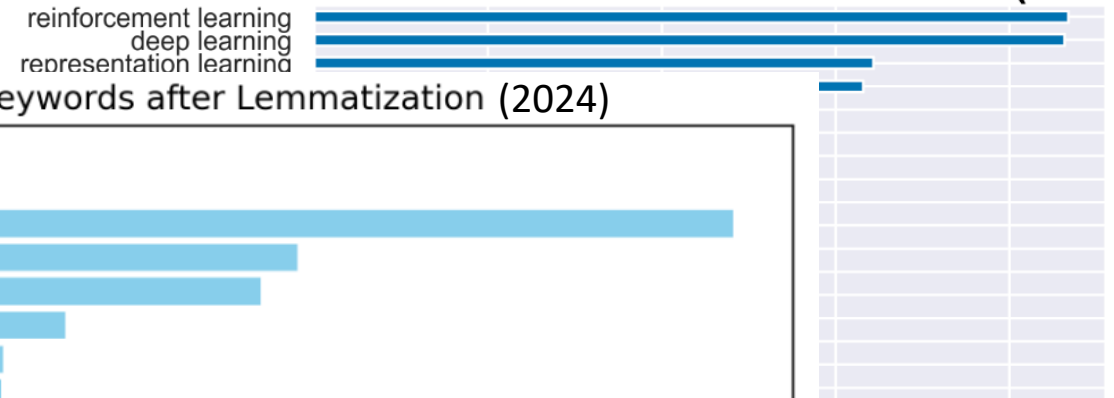
ICLR 2021 Submission Top 50 Keywords



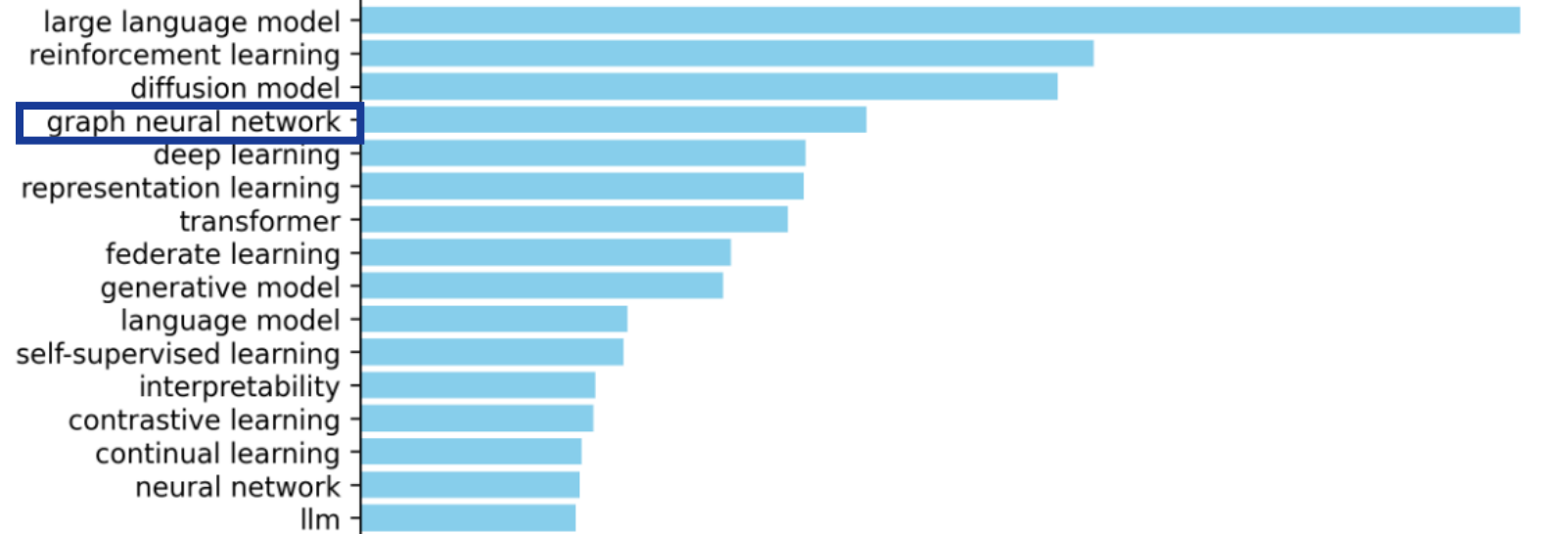
50 MOST APPEARED KEYWORDS (2022)



50 MOST APPEARED KEYWORDS (2023)

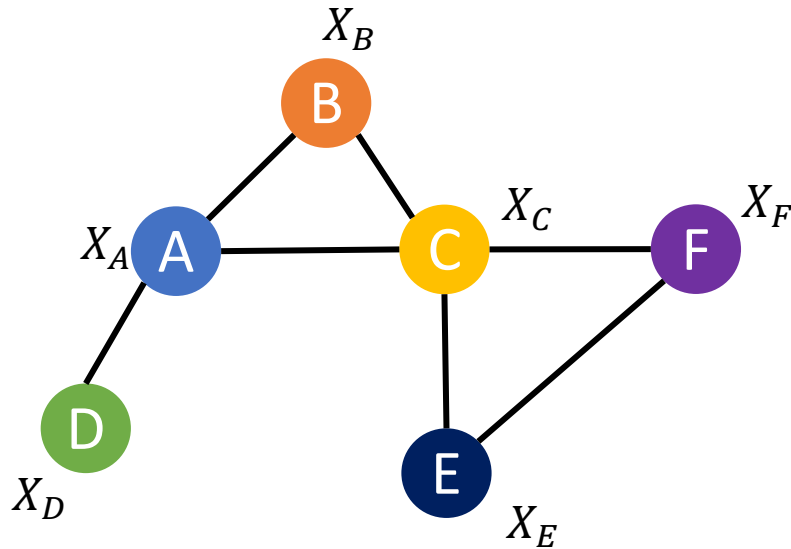


Top 50 Keywords after Lemmatization (2024)



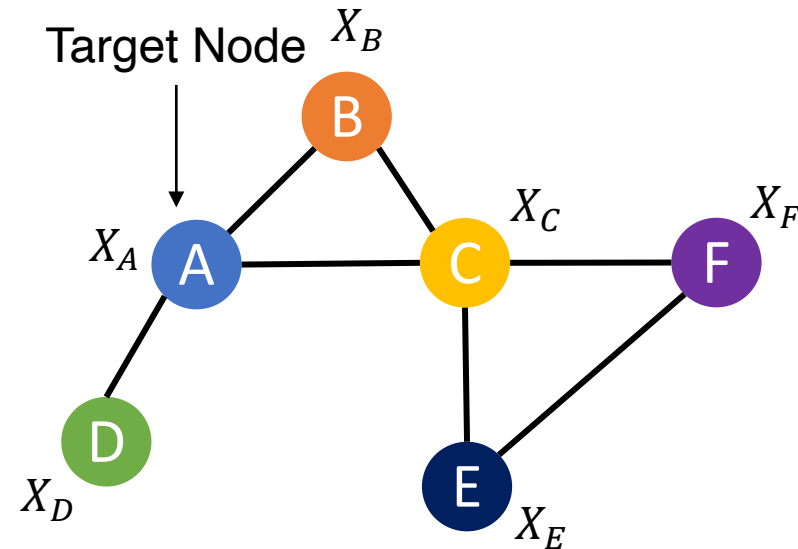
What is Graph Neural Network?

Problem definition



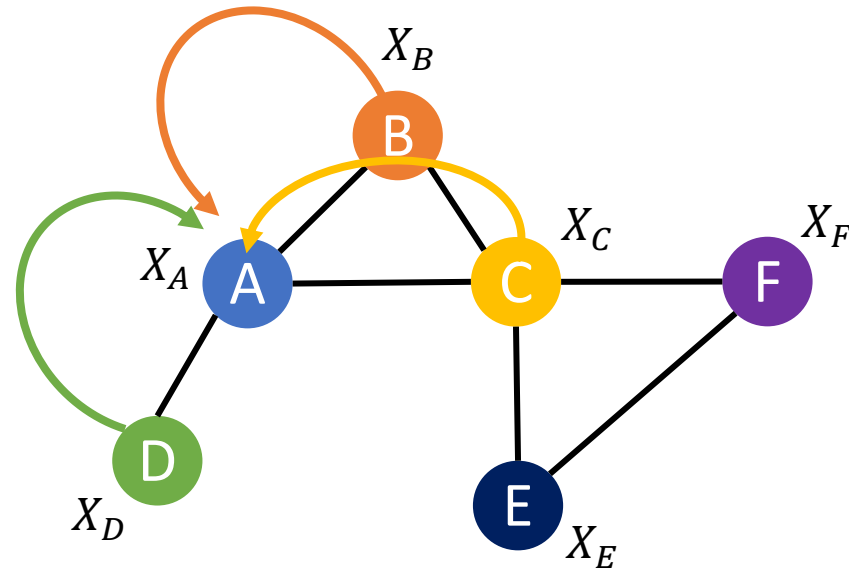
- **Given**
 - A graph
 - Node attributes
 - (part of nodes are labeled)
- **Find**
 - Node embeddings
- **Predict**
 - Labels for the remaining nodes

Graph Neural Networks



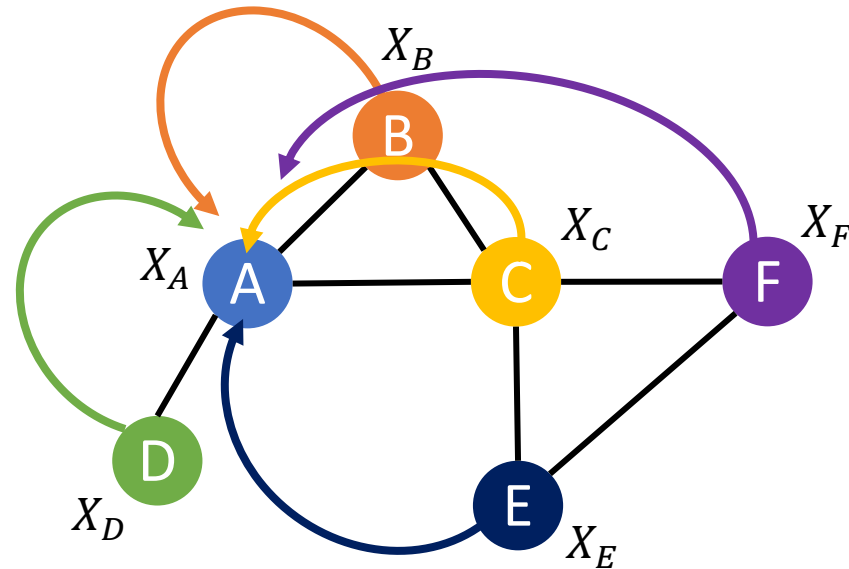
“Homophily: connected nodes are related/informative/similar”

Graph Neural Networks



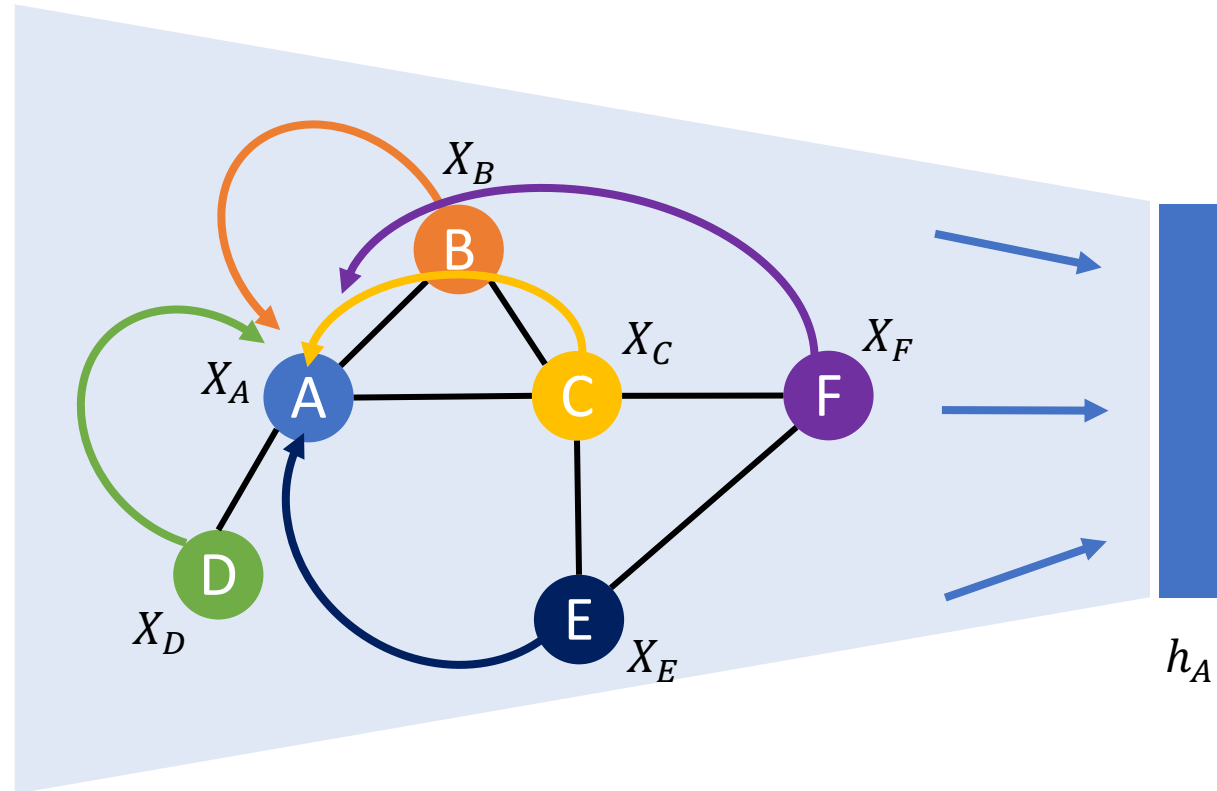
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Graph Neural Networks

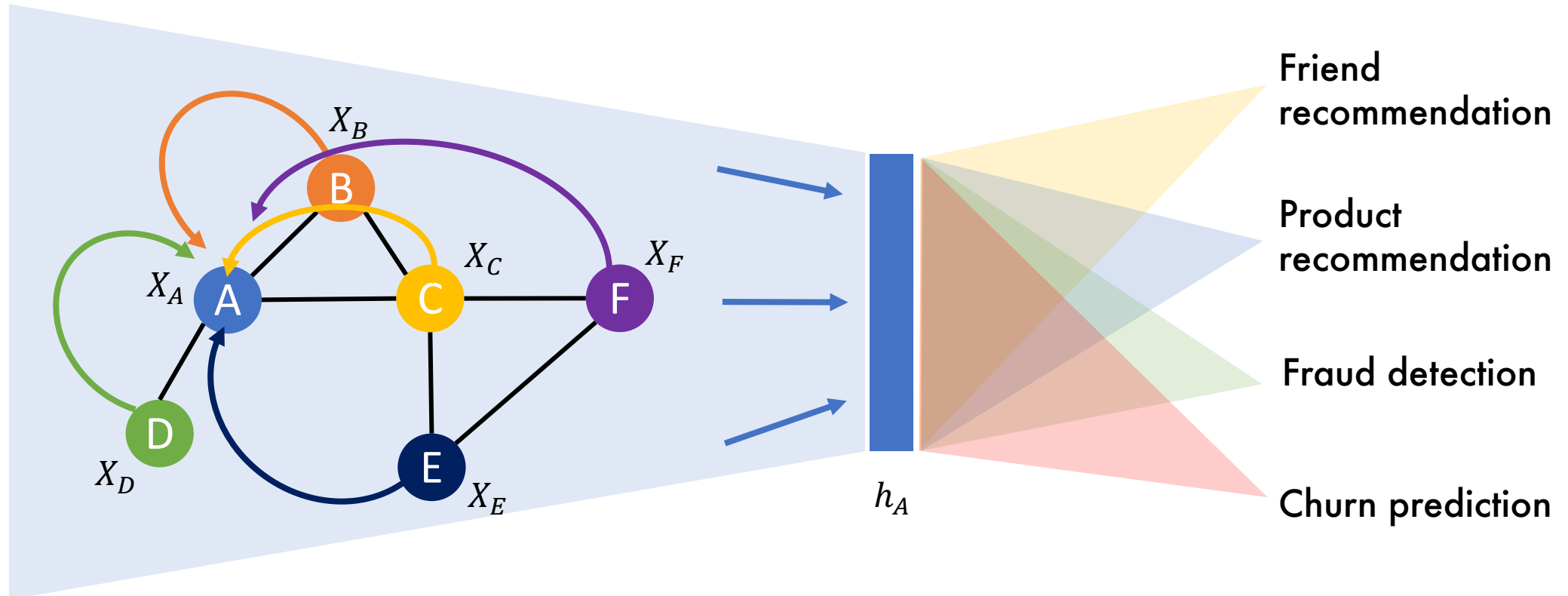


“Homophily: connected nodes are related/informative/similar”

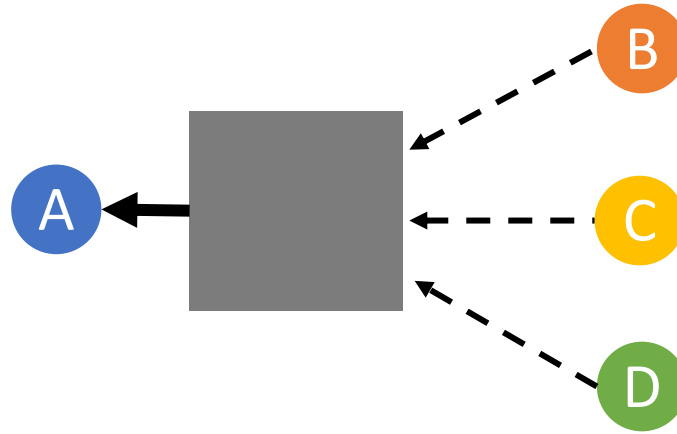
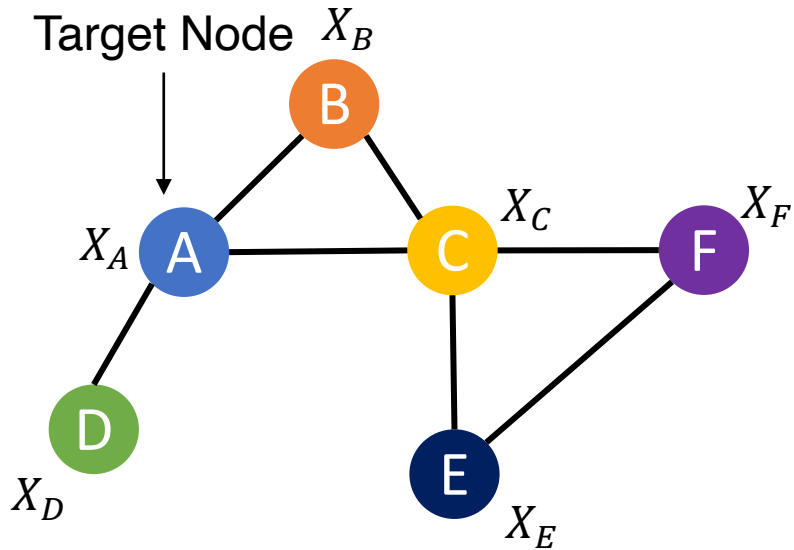
Graph Neural Networks



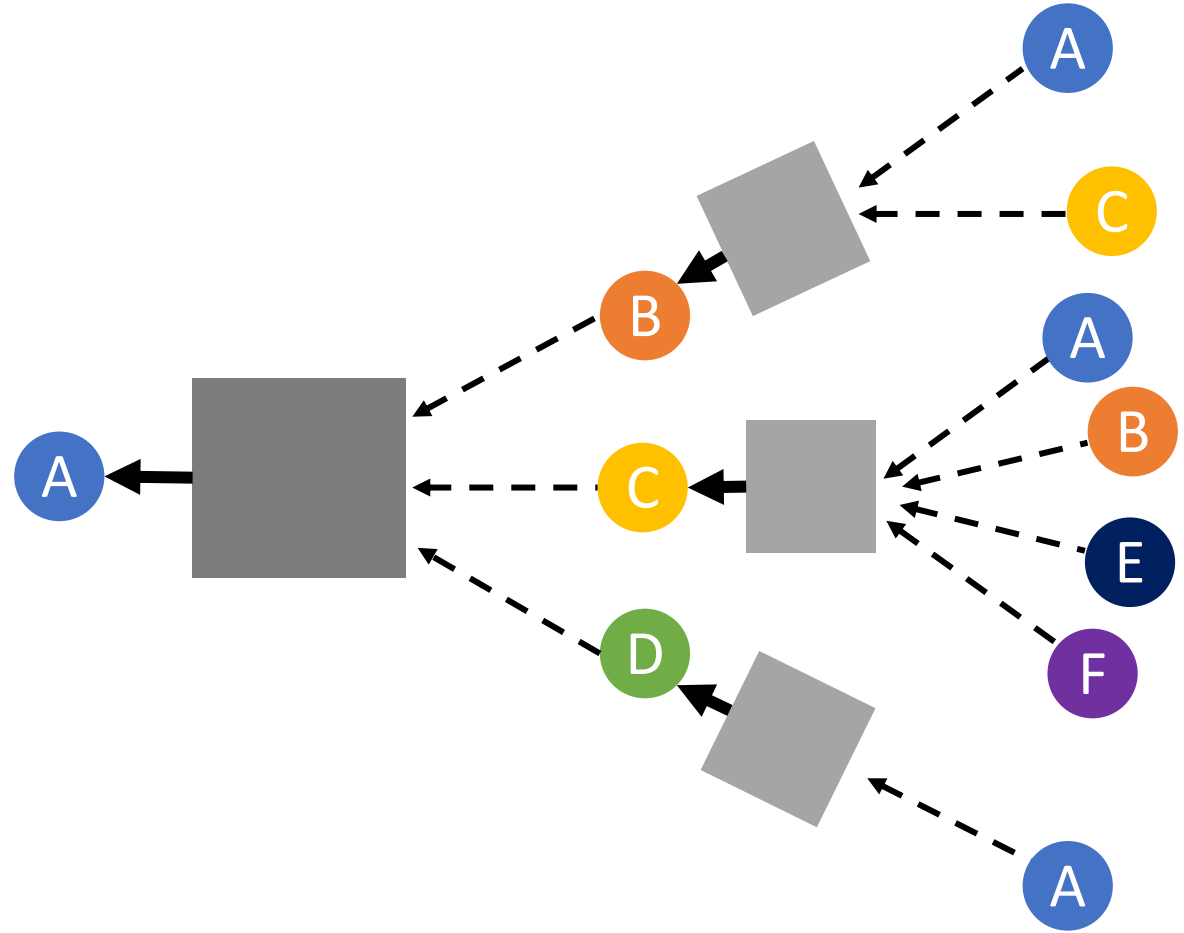
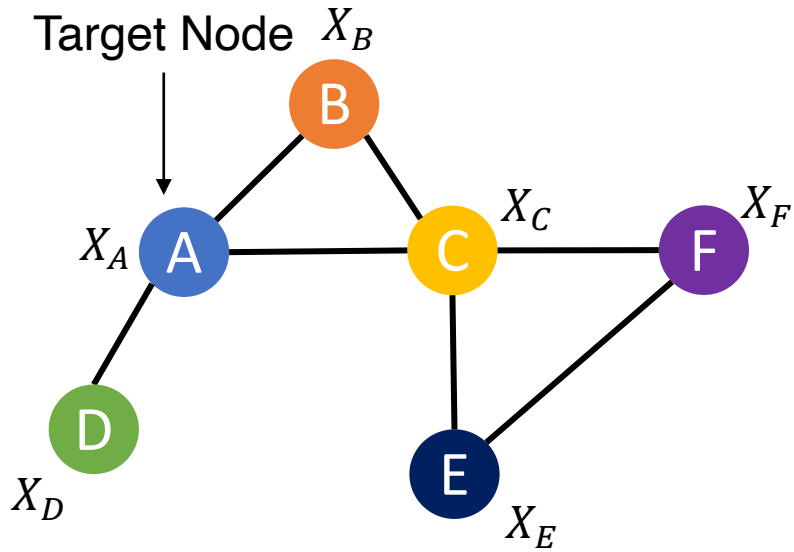
Graph Neural Networks



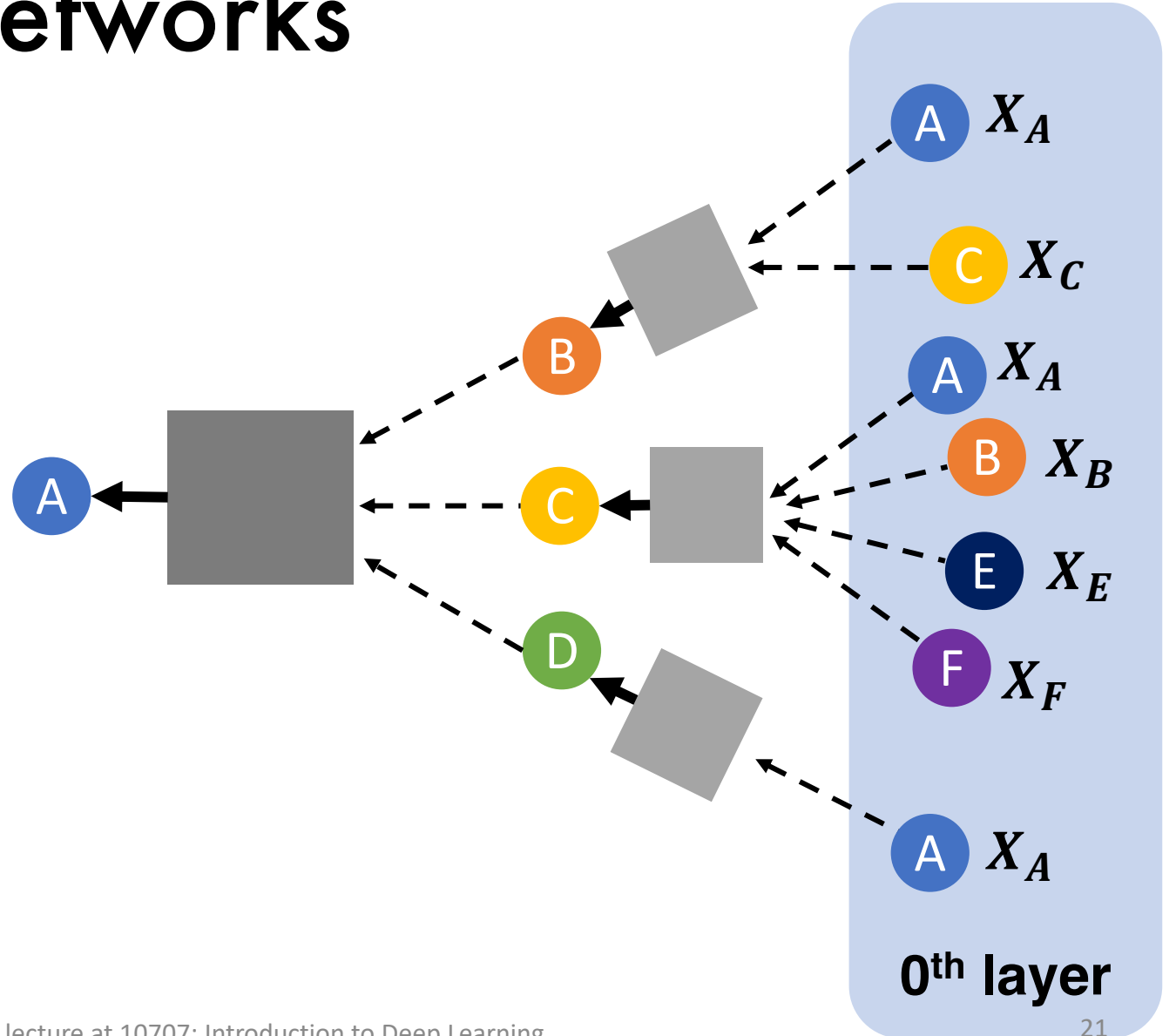
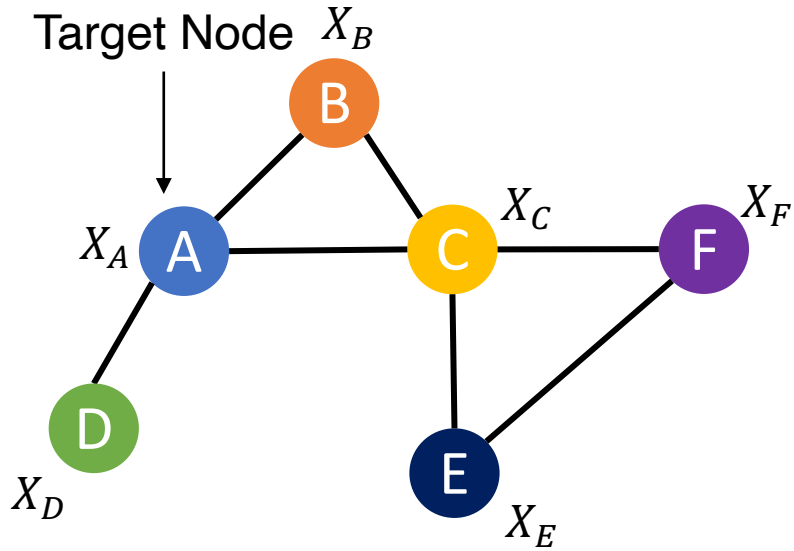
Graph Neural Networks



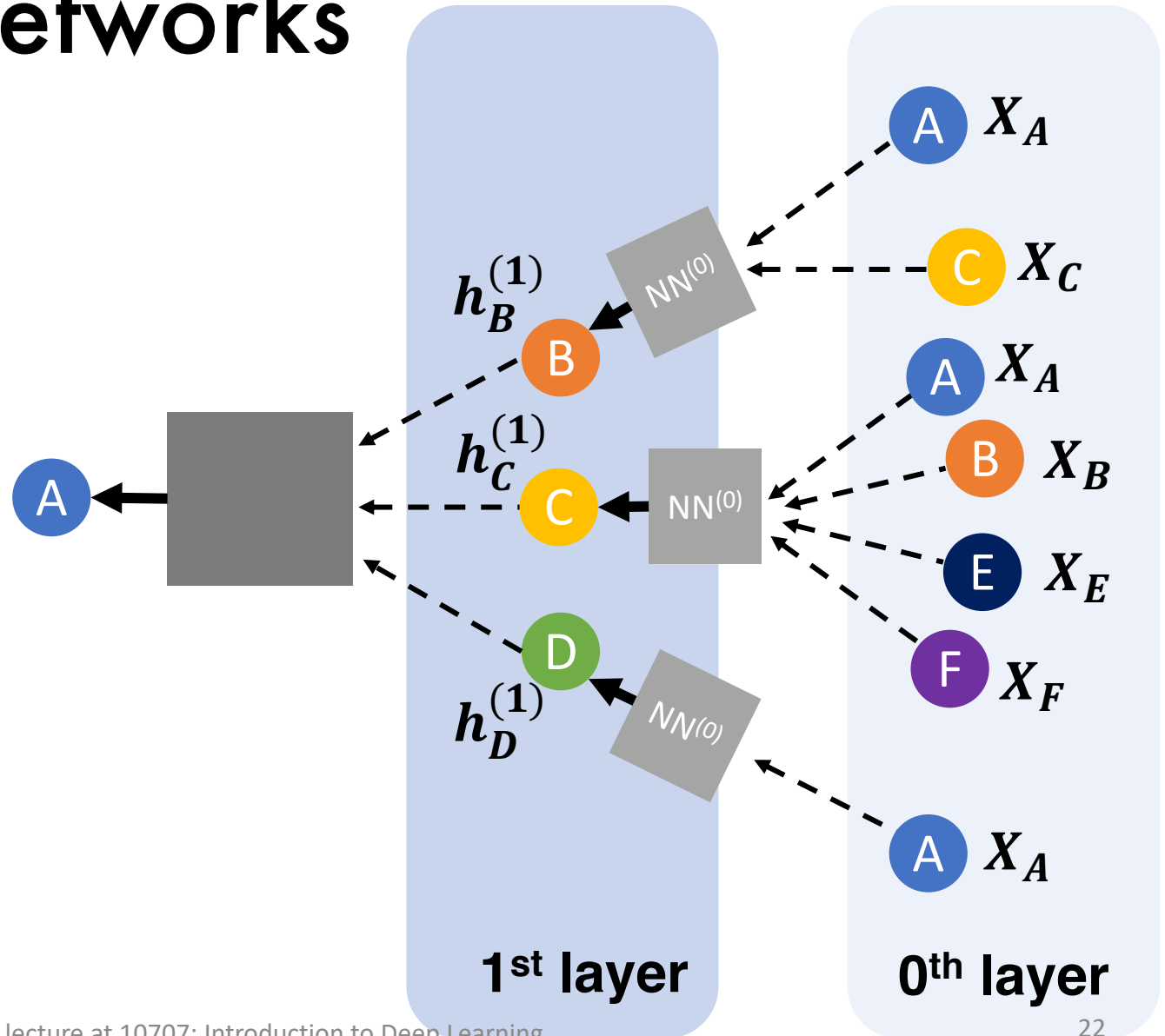
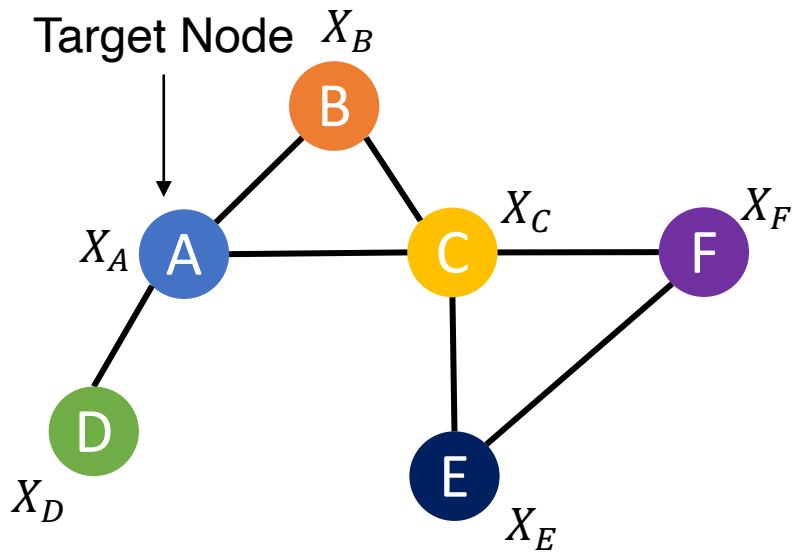
Graph Neural Networks



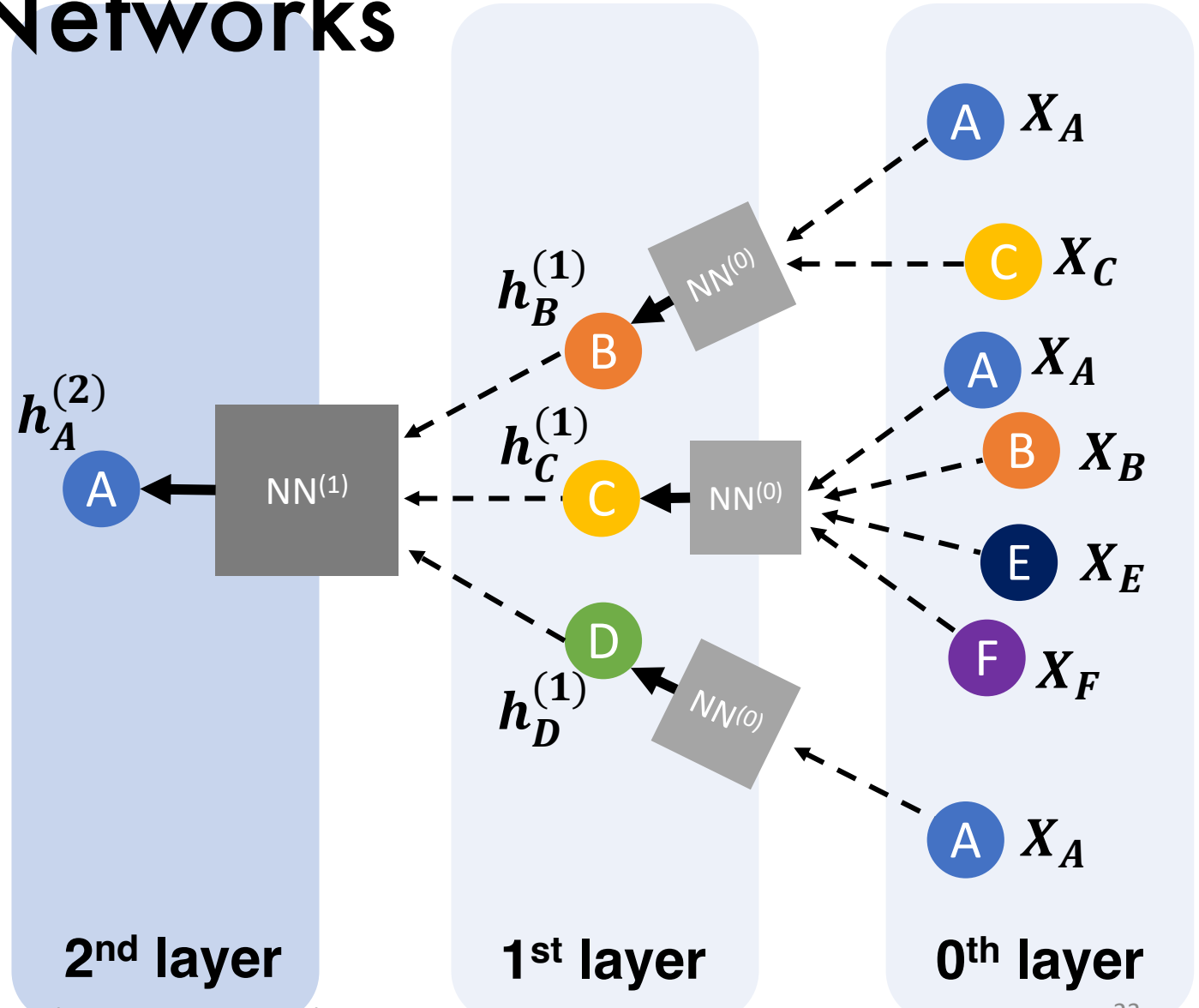
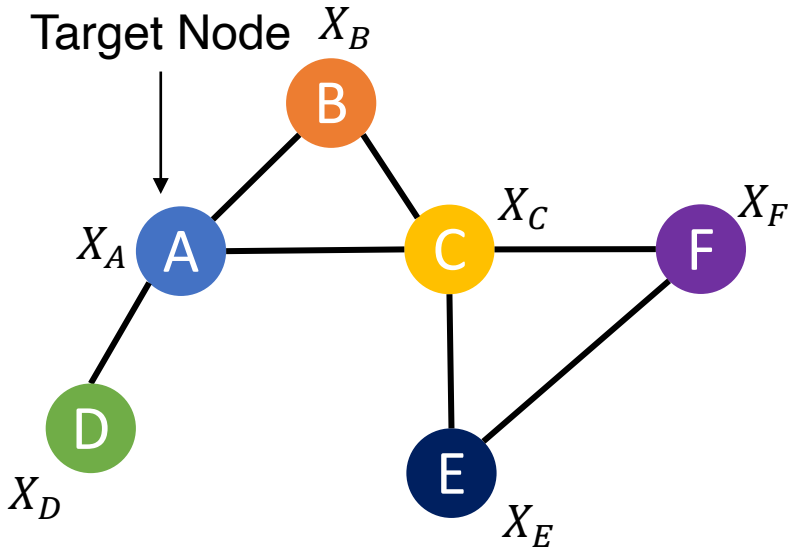
Graph Neural Networks



Graph Neural Networks



Graph Neural Networks



Graph Neural Networks

1. Aggregate messages from neighbors

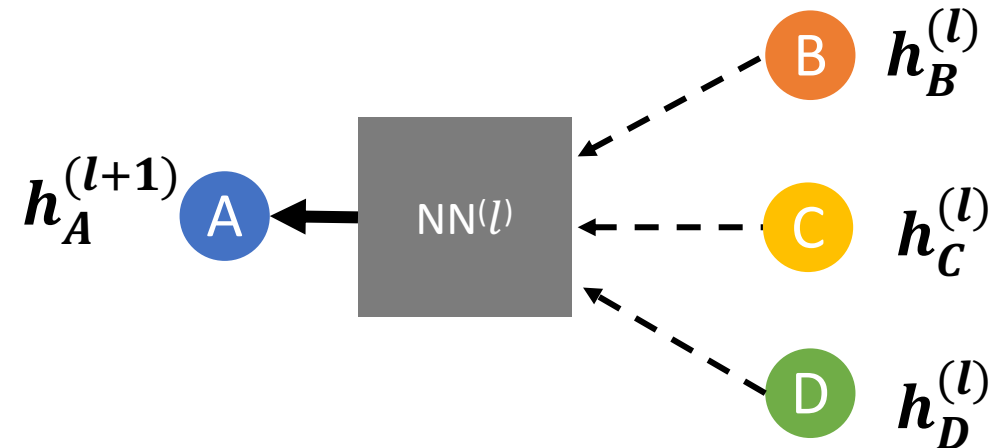
$h_v^{(l)}$: node embedding of v at l -th layer

$\mathcal{N}(v)$: neighboring nodes of v

$f^{(l)}$: aggregation function at l -th layer

$m_v^{(l)}$: message vector of v at l -th layer

$$\begin{aligned} m_A^{(l)} &= f^{(l)} \left(h_A^{(l)}, \{h_u^{(l)} : u \in \mathcal{N}(A)\} \right) \\ &= f^{(l)} \left(h_A^{(l)}, h_B^{(l)} h_C^{(l)} h_D^{(l)} \right) \end{aligned}$$



Neighbors of node A
 $\mathcal{N}(A) = \{B, C, D\}$

Graph Neural Networks

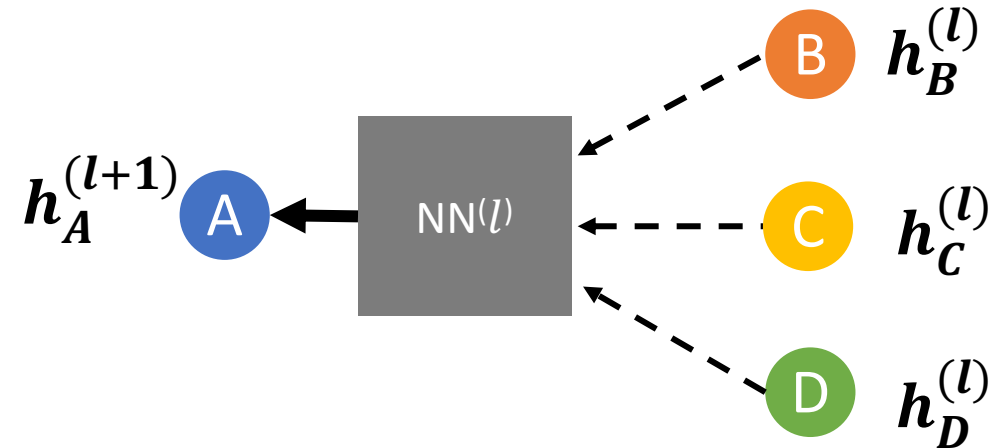
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2. Transform messages

$g^{(l)}$: transformation function at l -th layer

$$h_A^{(l+1)} = g^{(l)}(m_A^{(l)})$$



Neighbors of node A
 $\mathcal{N}(A) = \{B, C, D\}$

Graph Neural Networks

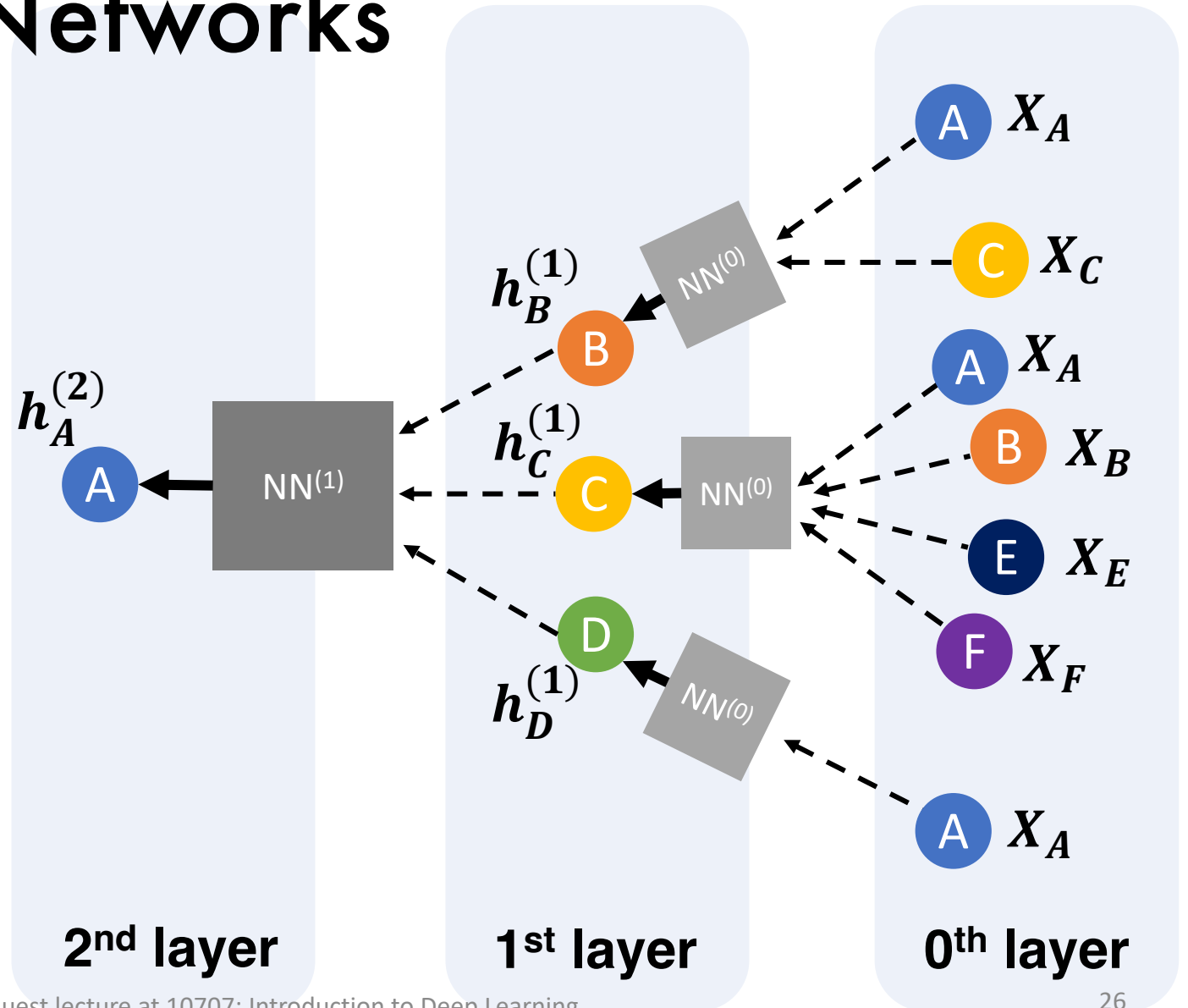
In each layer l ,
for each target node v :

1. Aggregate messages

$$m_v^{(l)} = f^{(l)} \left(h_v^{(l)}, \{h_u^{(l)} : u \in \mathcal{N}(v)\} \right)$$

2. Transform messages

$$h_v^{(l+1)} = g^{(l)}(m_v^{(l)})$$



Graph Neural Networks

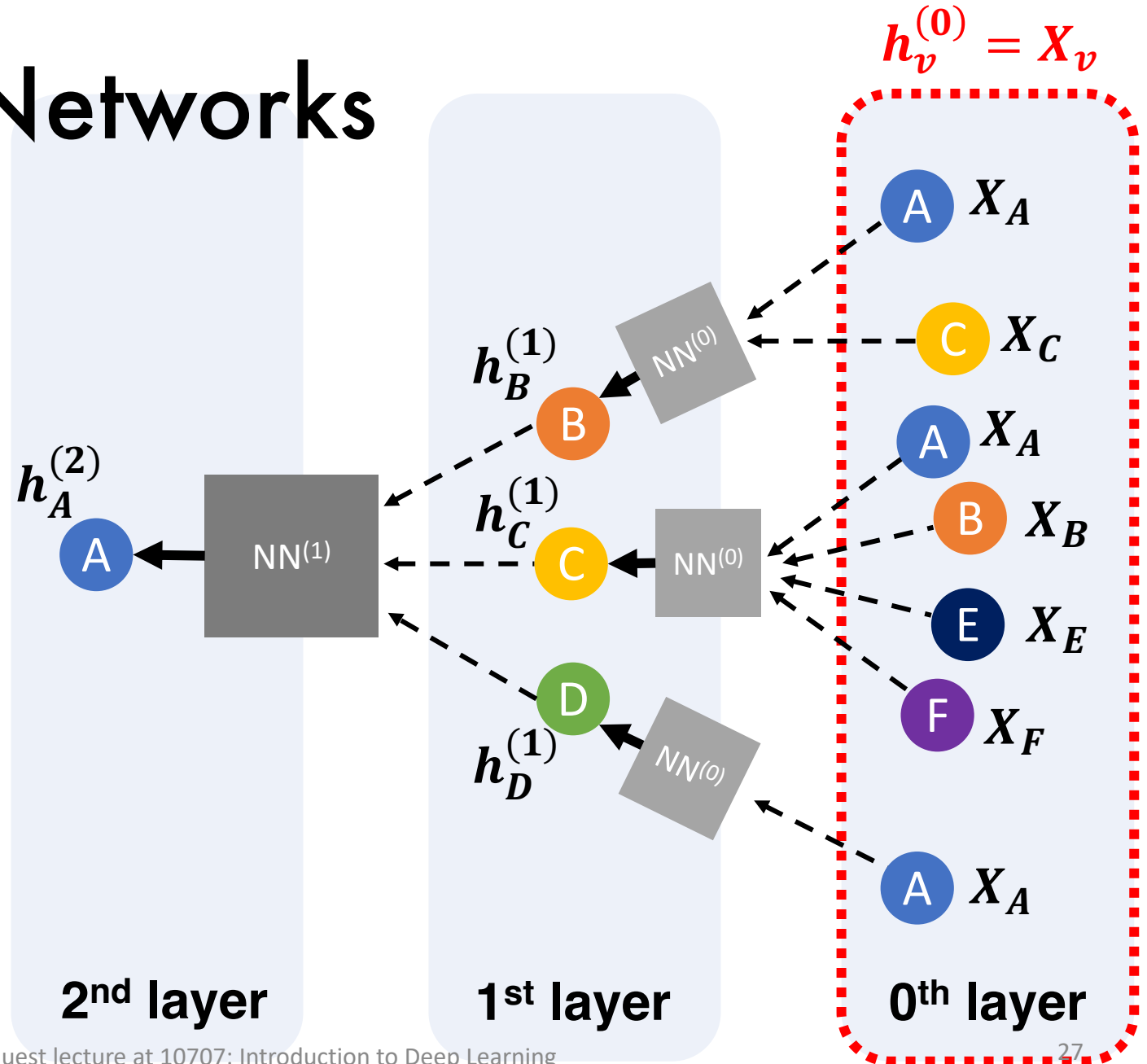
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Graph Neural Networks

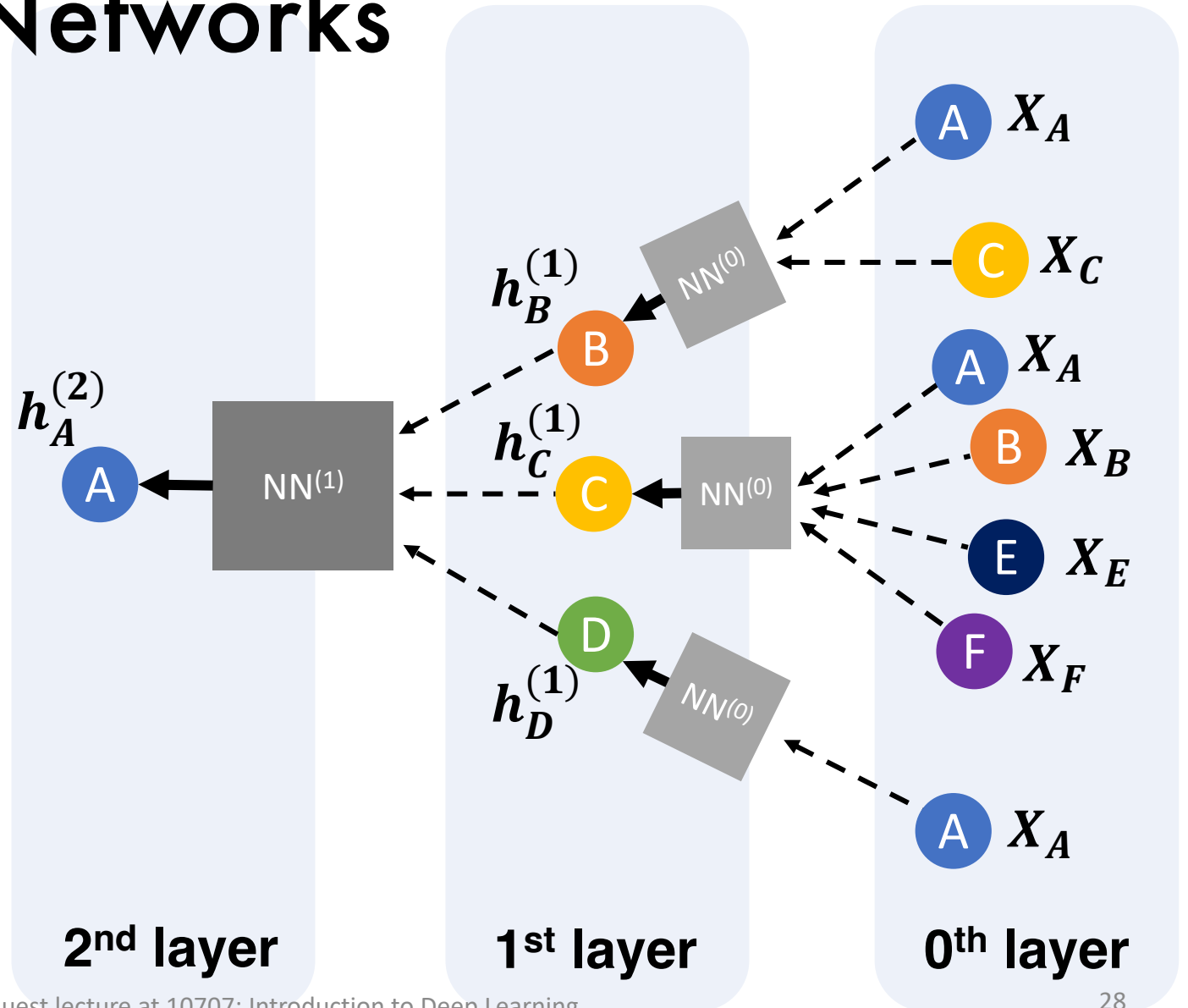
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2. Transform messages

$$h_v^{(l+1)} = \mathbf{g}^{(l)}(m_v^{(l)})$$



Graph Neural Networks

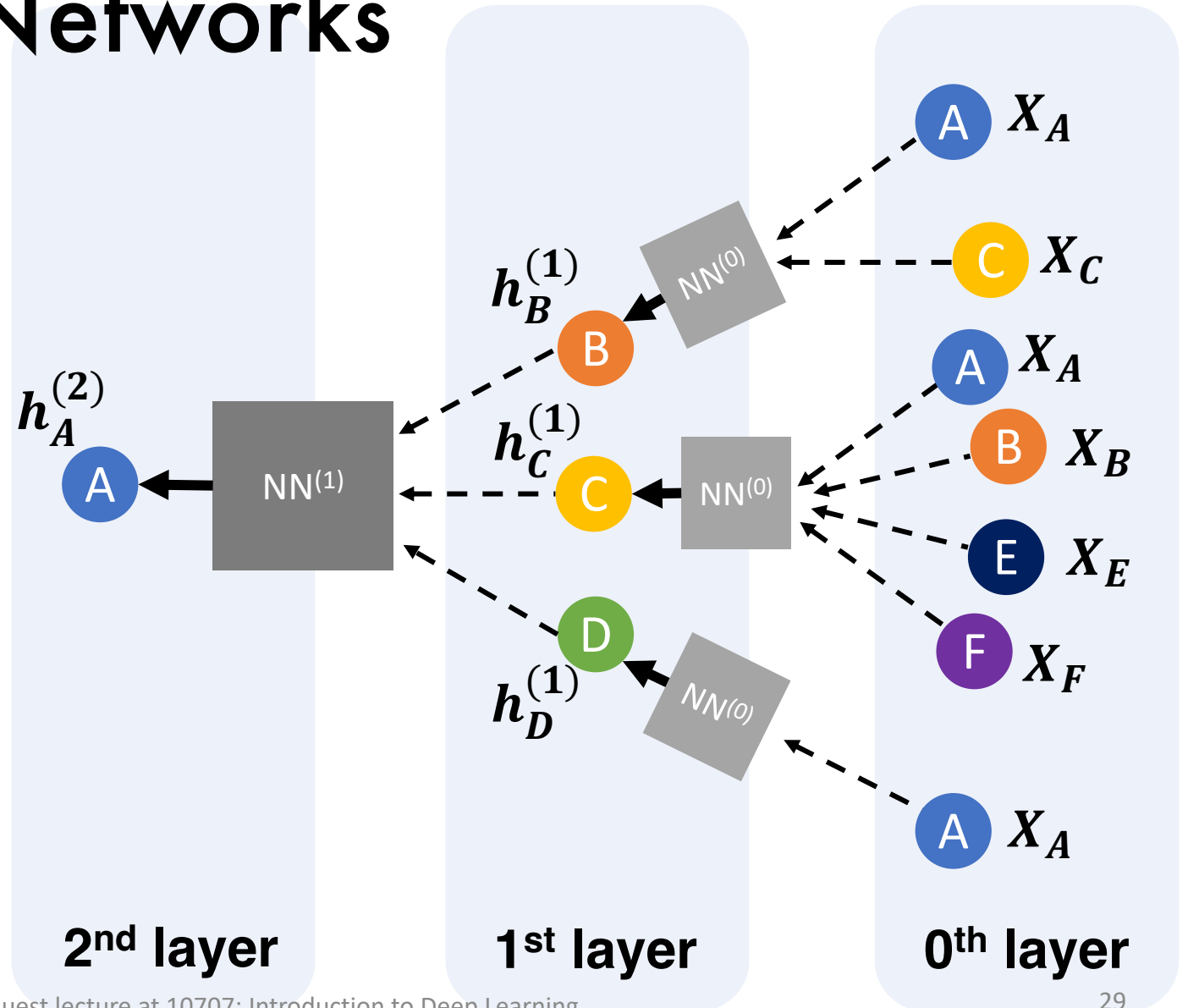
Graph Convolutional Networks^[1]

1. Aggregate messages

$$m_v^{(l)} = \frac{1}{|\mathcal{N}(v) + 1|} \sum_{u \in \mathcal{N}(v) \cup \{v\}} h_u^{(l)}$$

2. Transform messages

$$h_v^{(l+1)} = \sigma(W^{(l)} \circ m_v^{(l)})$$



[1] Kipf, Thomas N., et al. "Semi-supervised classification with graph convolutional networks."

Graph Neural Networks

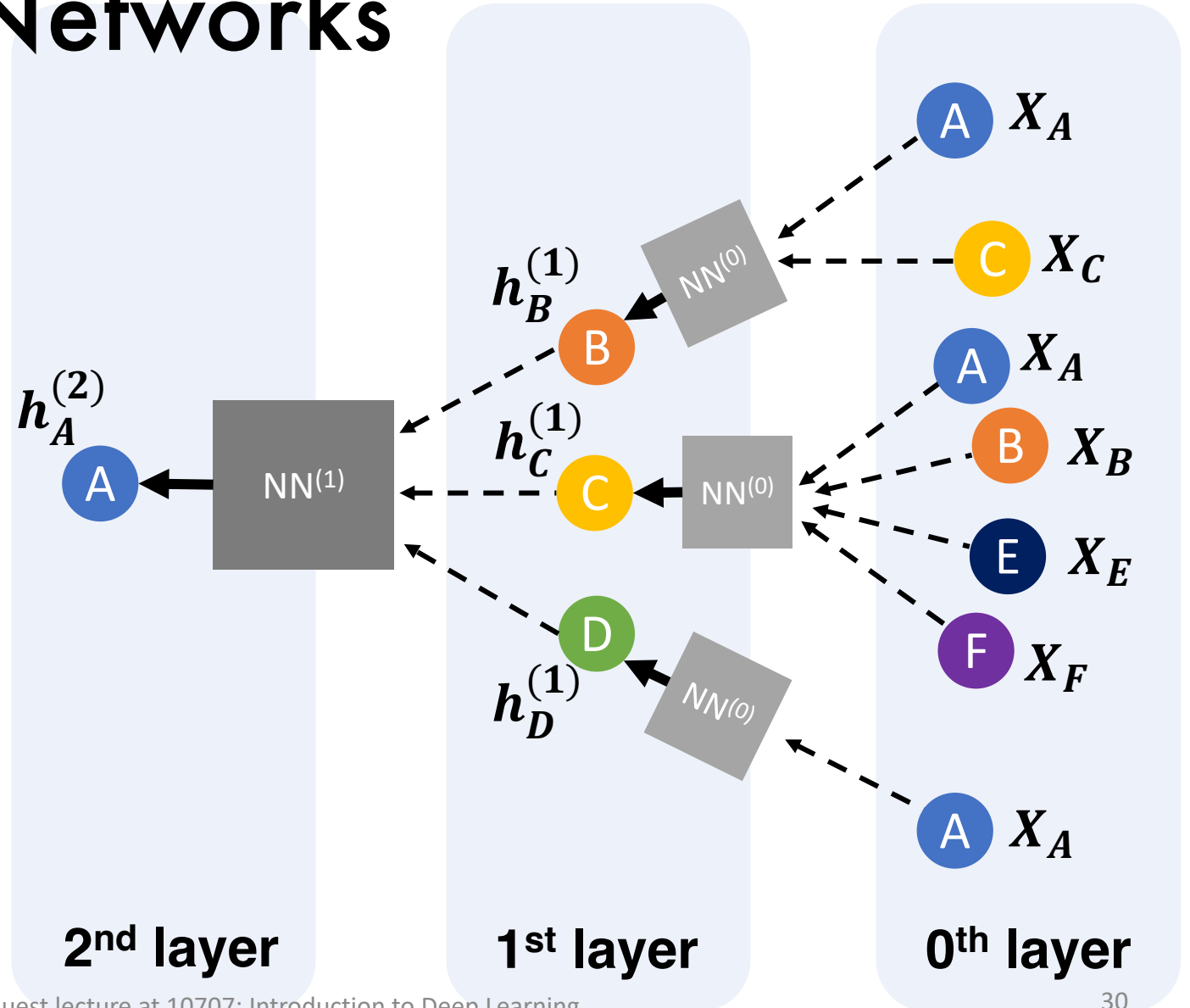
Graph Isomorphism Networks^[2]

1. Aggregate messages

$$m_v^{(l)} = \sum_{u \in \mathcal{N}(v) \cup \{v\}} h_u^{(l)}$$

2. Transform messages

$$h_v^{(l+1)} = \sigma(W^{(l)} \circ m_v^{(l)})$$



[2] Xu, Keyulu, et al. "How powerful are graph neural networks?."

Graph Neural Networks

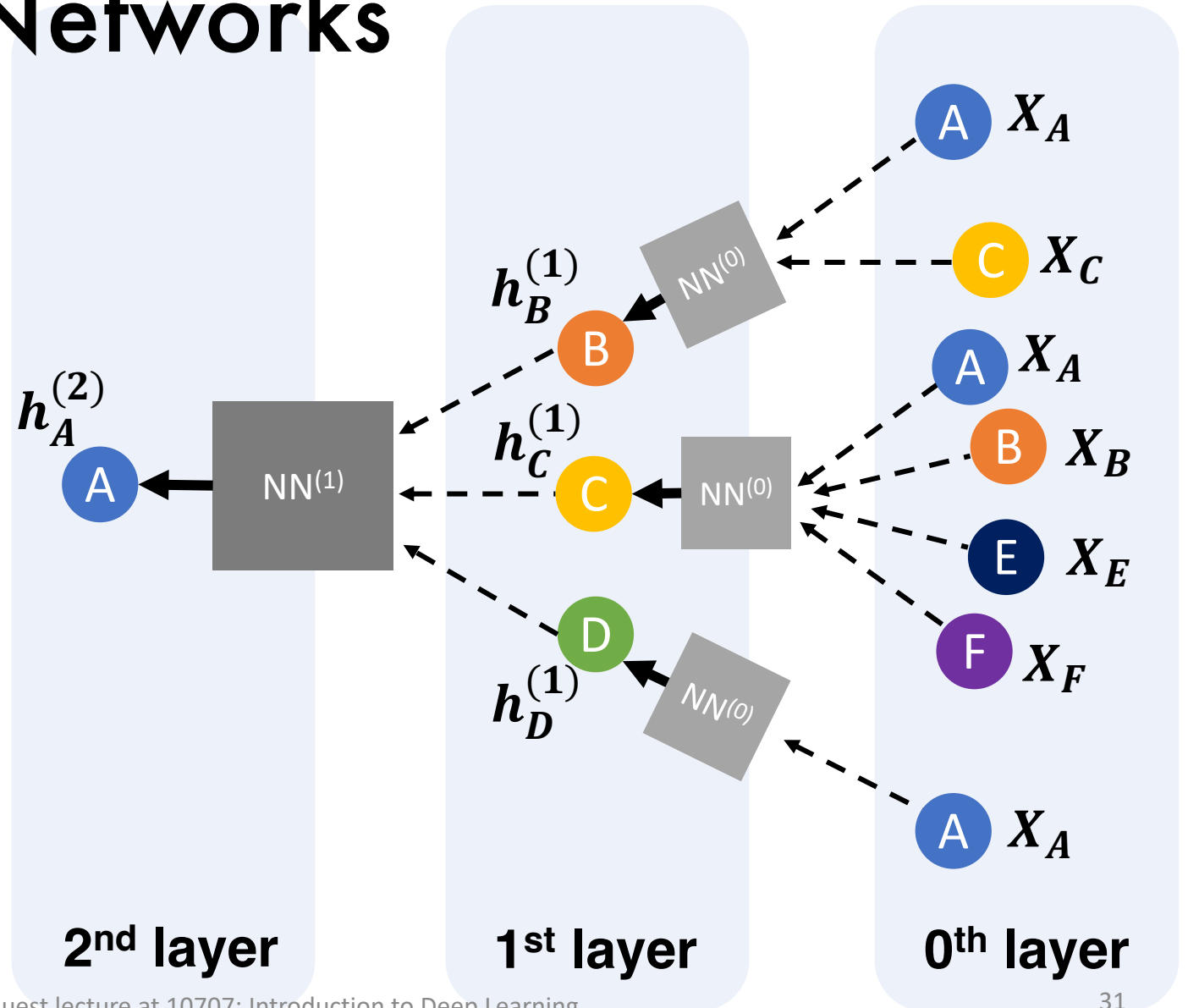
Simplified GCN^[3]

1. Aggregate messages

$$m_v^{(l)} = \frac{1}{|\mathcal{N}(v) + 1|} \sum_{u \in \mathcal{N}(v) \cup \{v\}} h_u^{(l)}$$

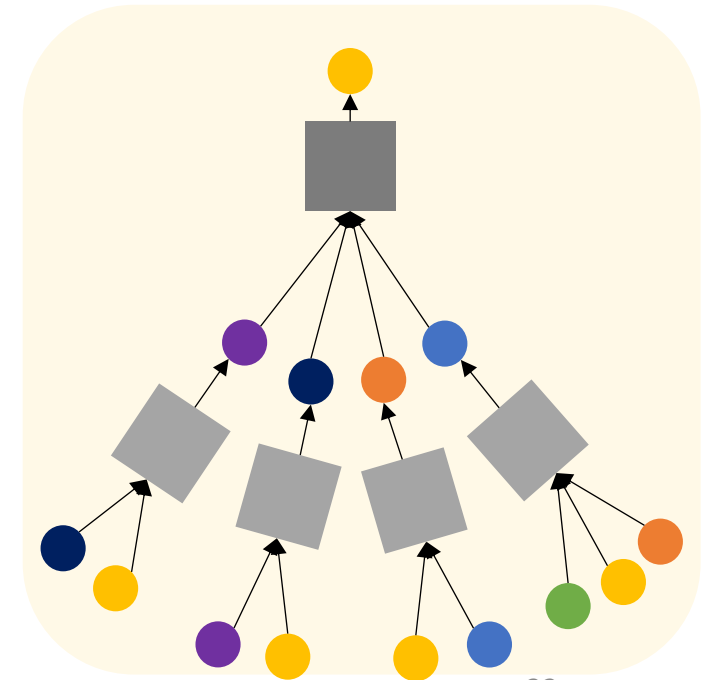
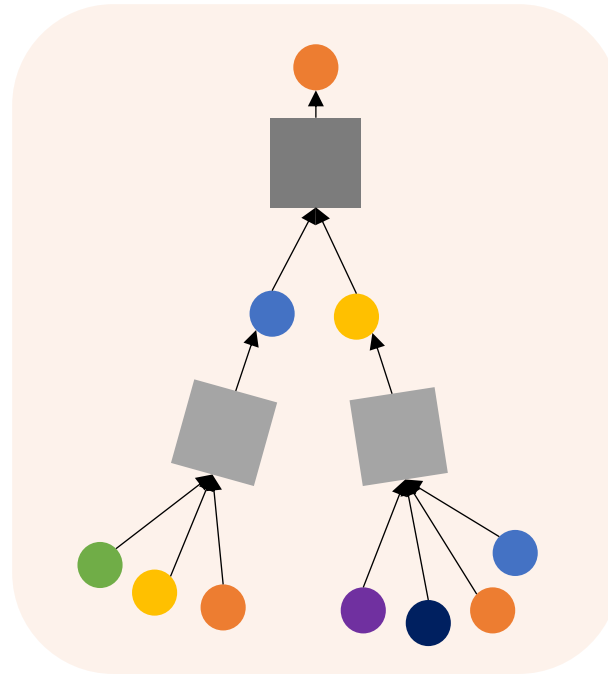
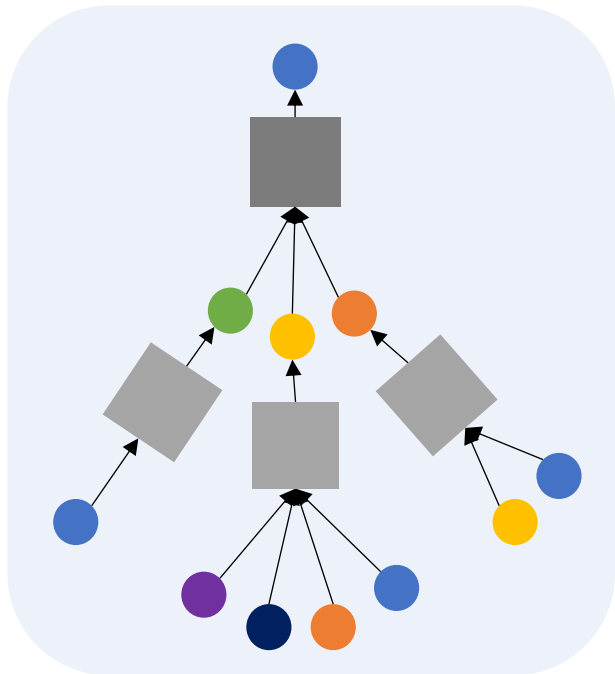
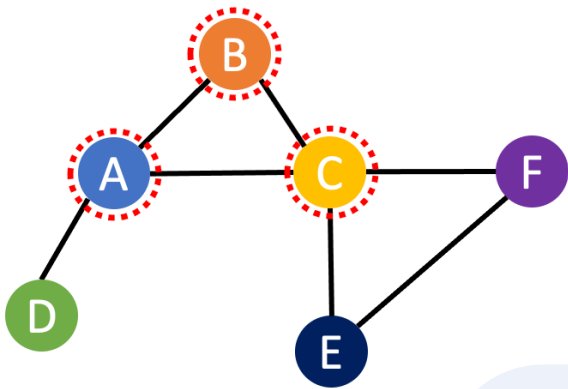
2. Transform messages

$$h_v^{(l+1)} = W^{(l)} \circ m_v^{(l)}$$

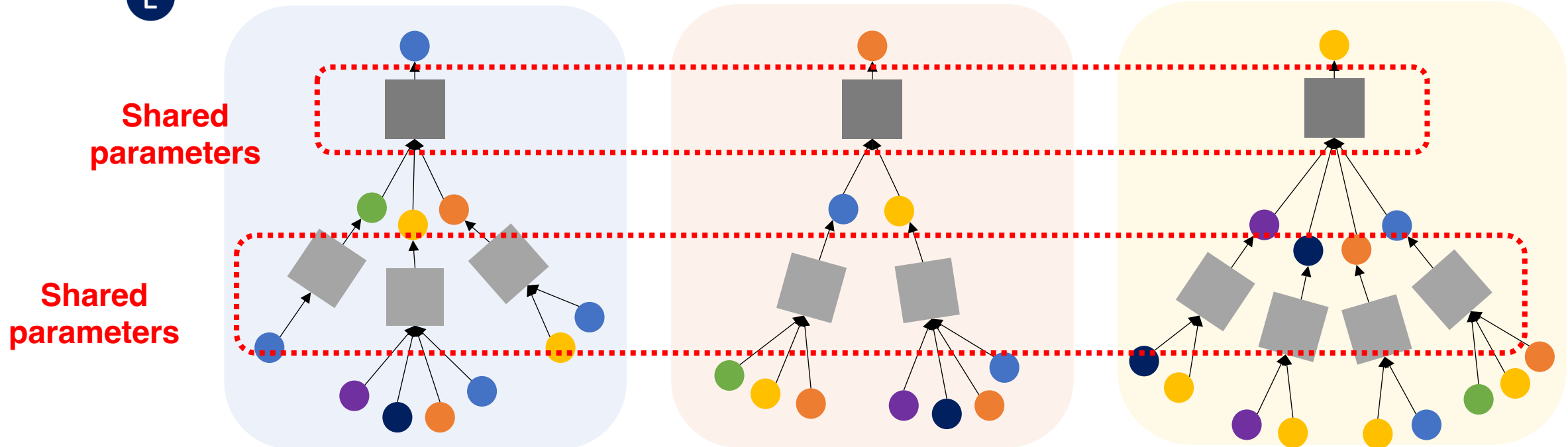
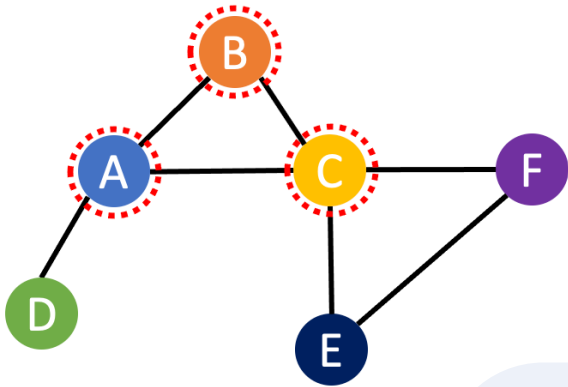


[3] Wu, Felix, et al. "Simplifying graph convolutional networks."

Computation graphs

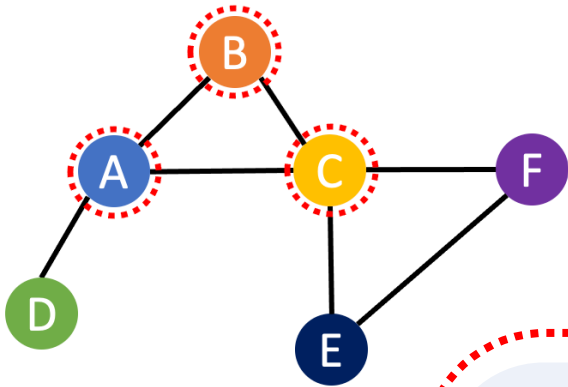


Computation graphs

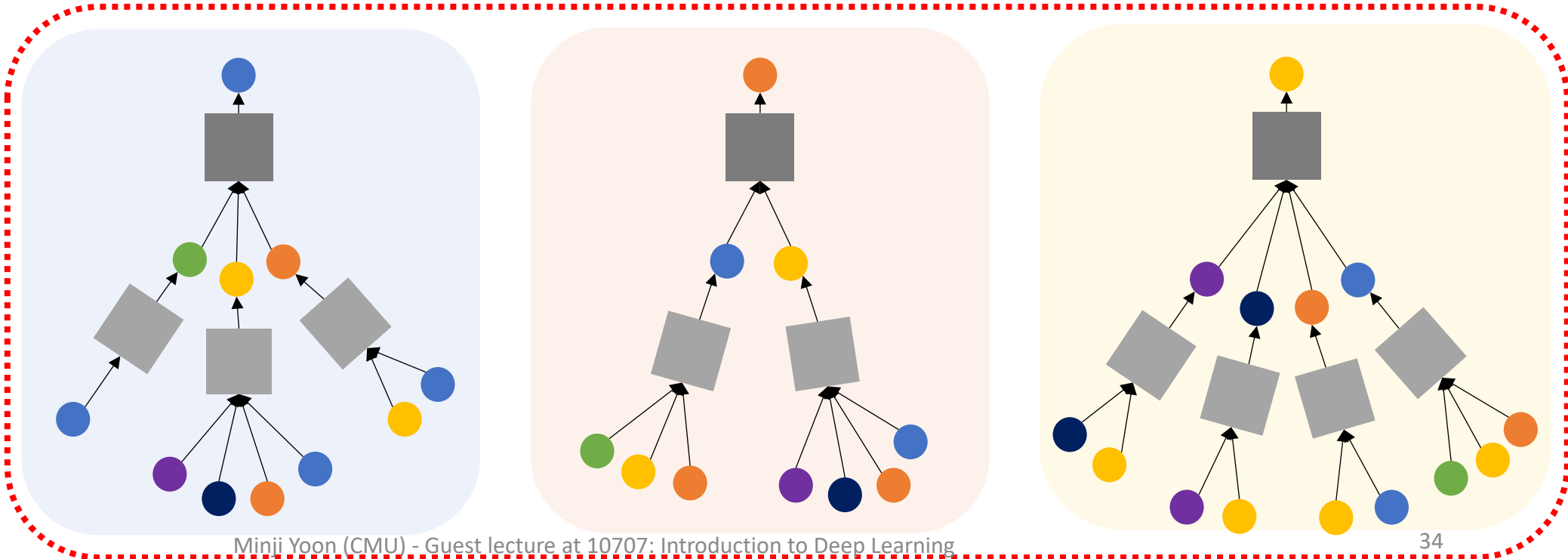


Batch execution

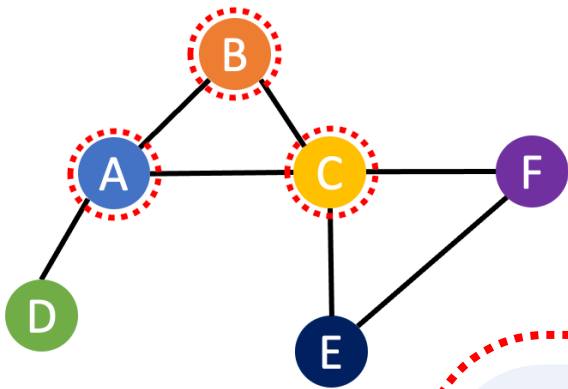
$$h_v^{(l)} = \sigma(W^{(l)} \circ \left(\frac{1}{|\mathcal{N}(v)+1|} \sum_{u \in \mathcal{N}(v) \cup \{v\}} h_u^{(l-1)} \right))$$



Batch size = 3



Batch execution



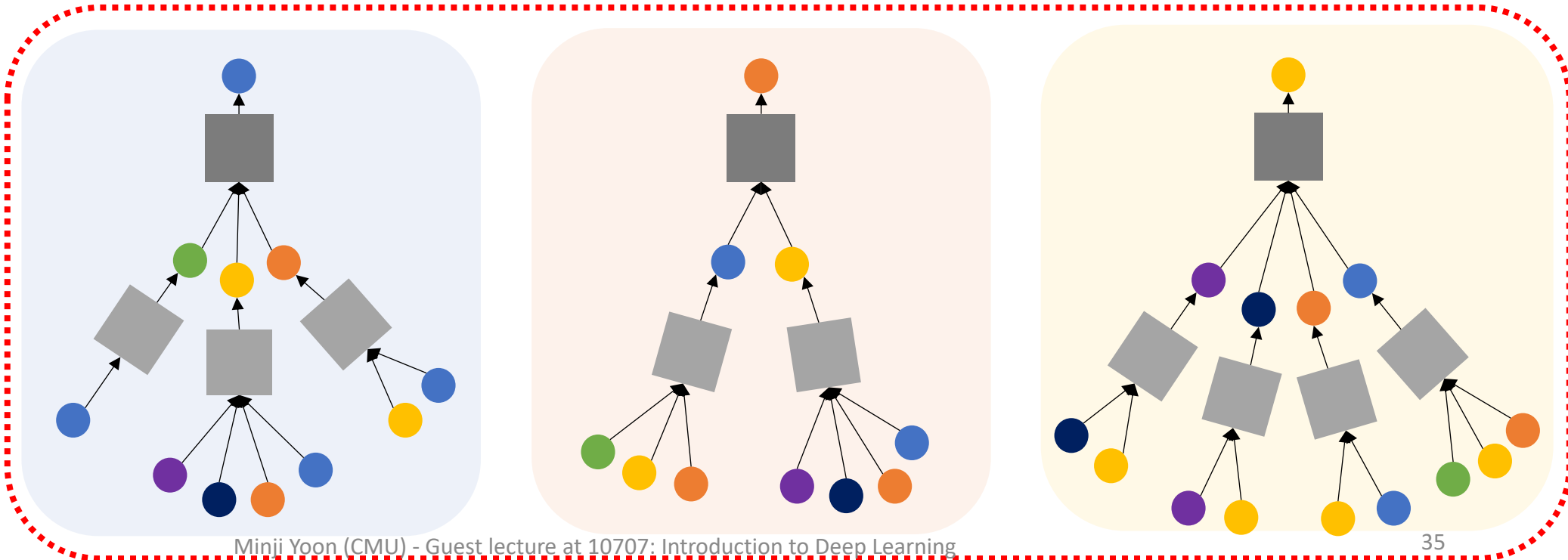
$$h_v^{(l)} = \sigma(W^{(l)} \circ \left(\frac{1}{|\mathcal{N}(v)+1|} \sum_{u \in \mathcal{N}(v) \cup \{v\}} h_u^{(l-1)} \right))$$

$$\mathbf{H}^{(l)} = \sigma(\widetilde{(\mathbf{A} + \mathbf{I})} \mathbf{H}^{(l-1)} \mathbf{W}^{(l)})$$

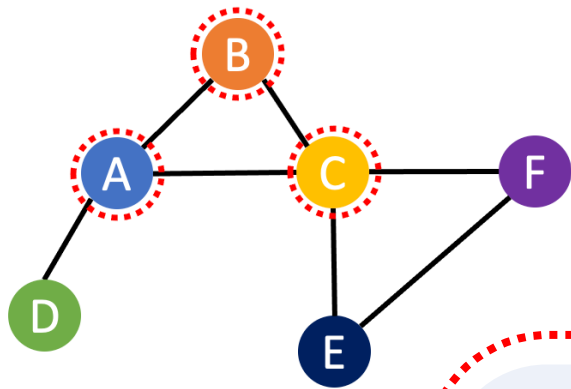
Node embedding matrix

(row-normalized) Adjacency matrix

Batch size = 3



Batch execution



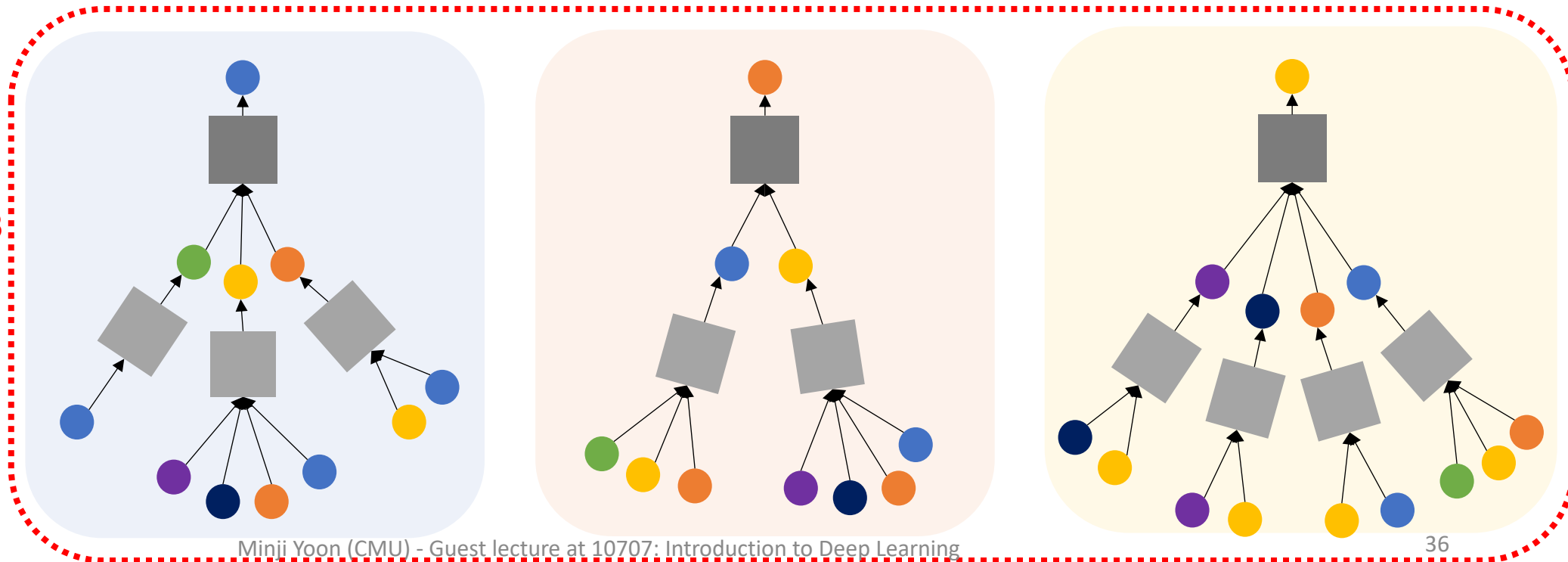
$$h_v^{(l)} = \sigma(W^{(l)} \circ \left(\frac{1}{|\mathcal{N}(v)+1|} \sum_{u \in \mathcal{N}(v) \cup \{v\}} h_u^{(l-1)} \right))$$

$$H^{(l)} = \sigma(\widetilde{(A+I)} H^{(l-1)} W^{(l)})$$

Fixed

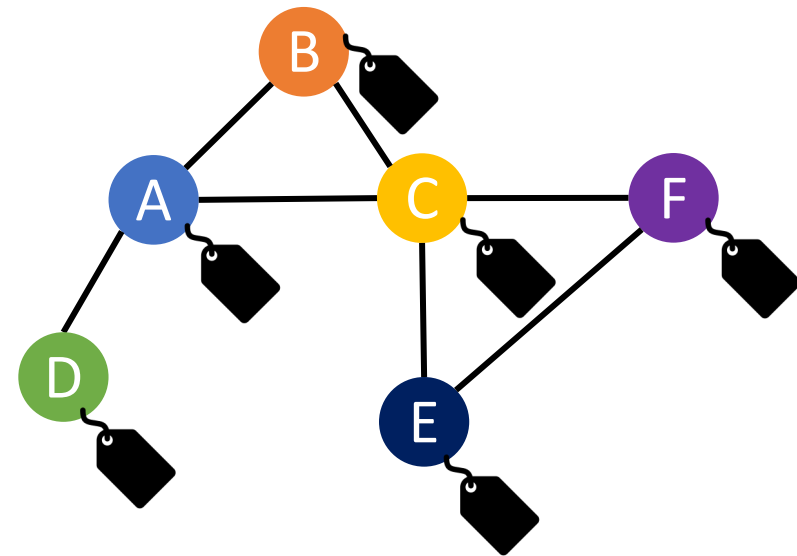
Trainable

Batch size = 3



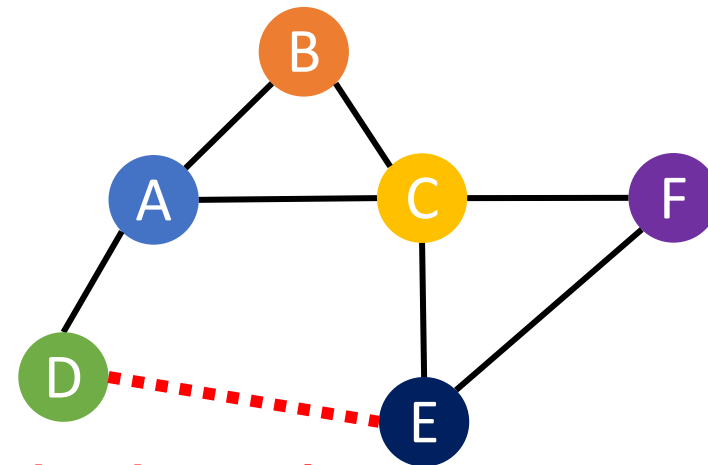
Downstream tasks

- Node-level prediction



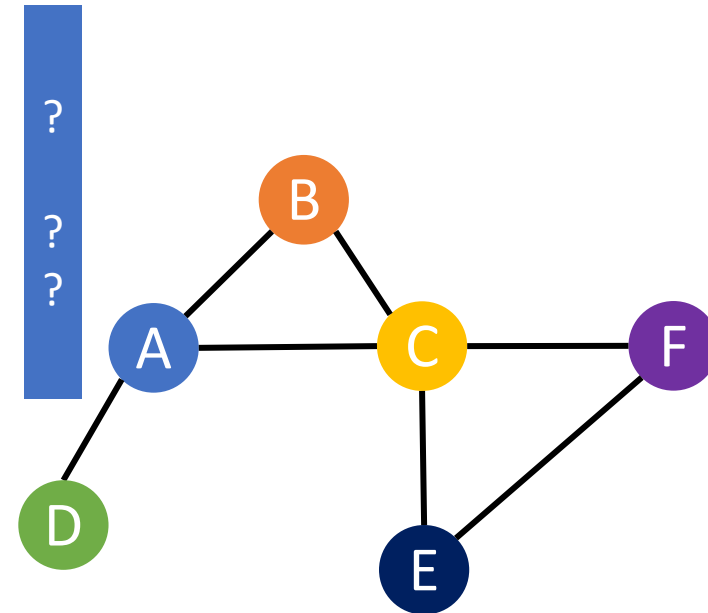
Downstream tasks

- Node-level prediction
- Edge-level prediction



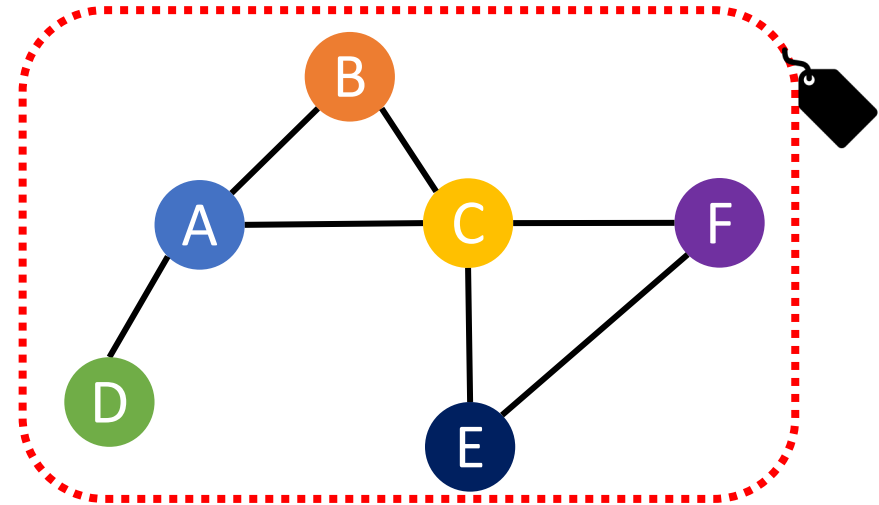
Downstream tasks

- Node-level prediction
- Edge-level prediction
- Attribute-level prediction



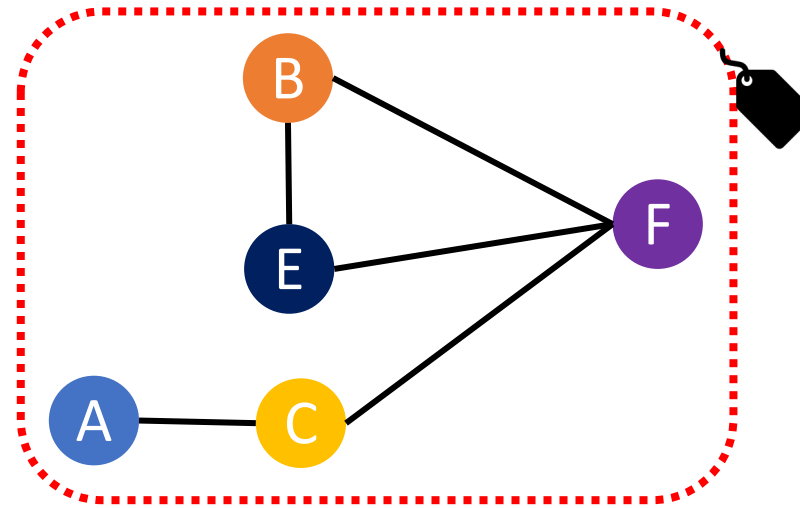
Downstream tasks

- Node-level prediction
- Edge-level prediction
- Attribute-level prediction
- Graph-level prediction

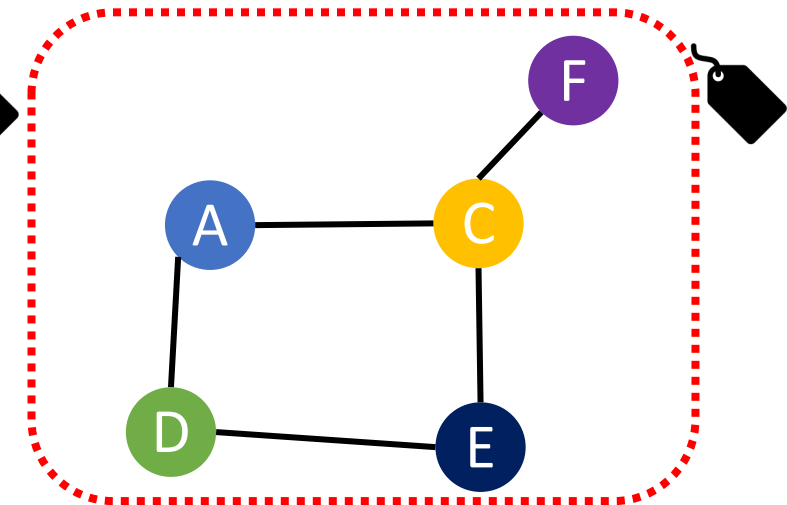
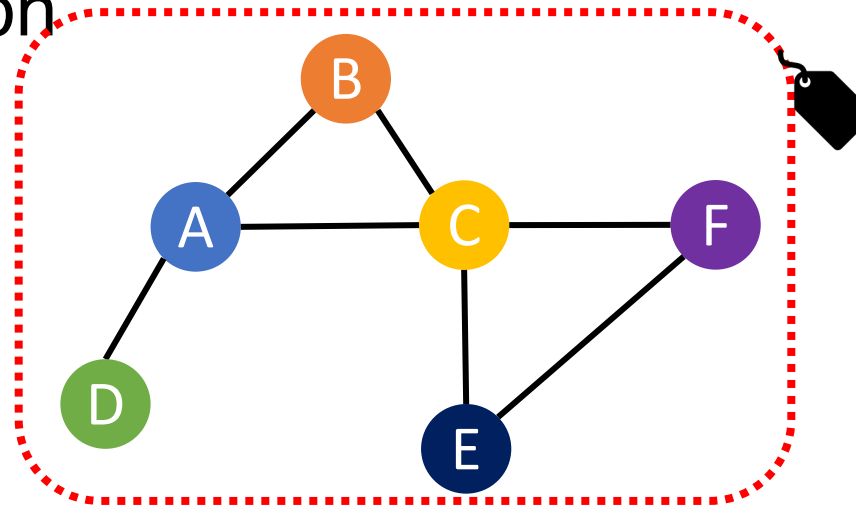


Downstream tasks

- Node-level prediction
- Edge-level prediction
- Attribute-level prediction
- Graph-level prediction

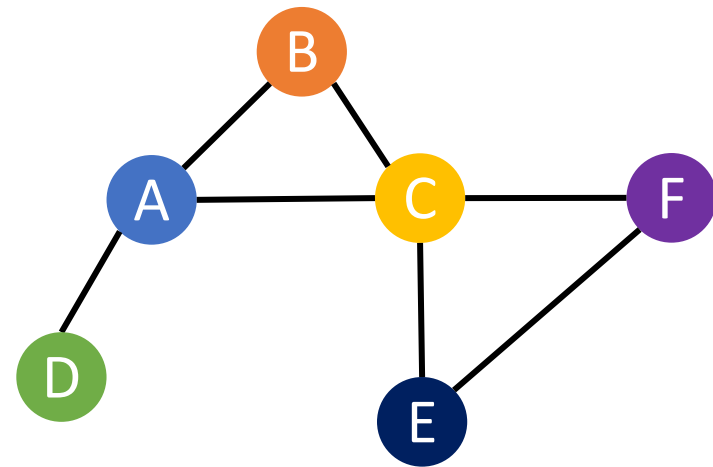


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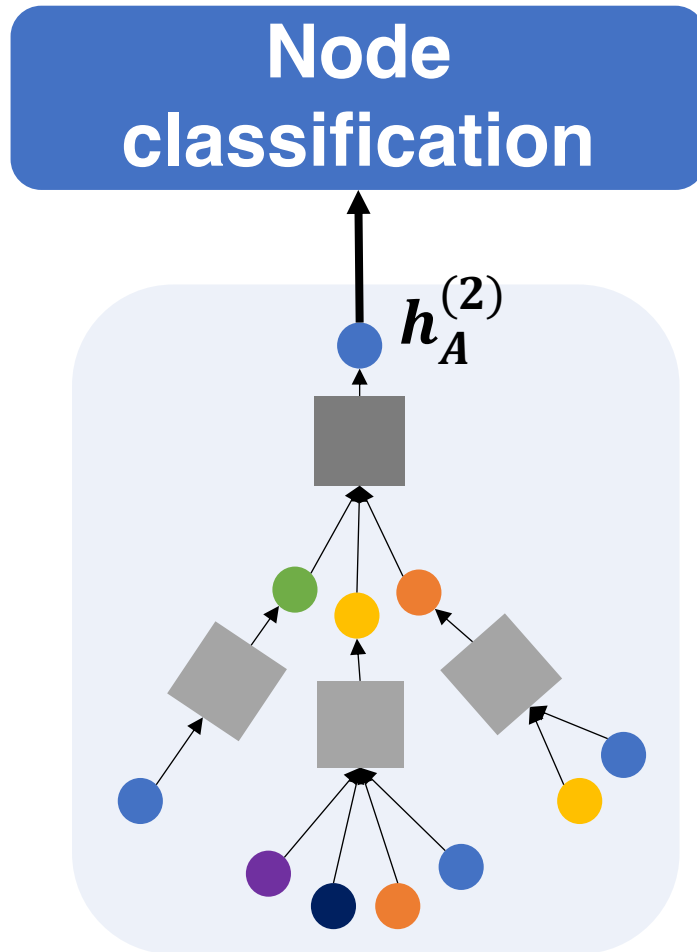


Downstream tasks

- **Node-level prediction**
- Edge-level prediction
- Attribute-level prediction
- **Graph-level prediction**

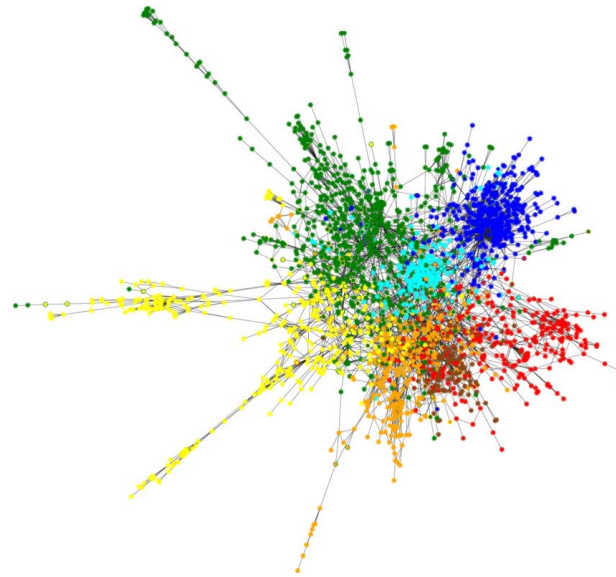
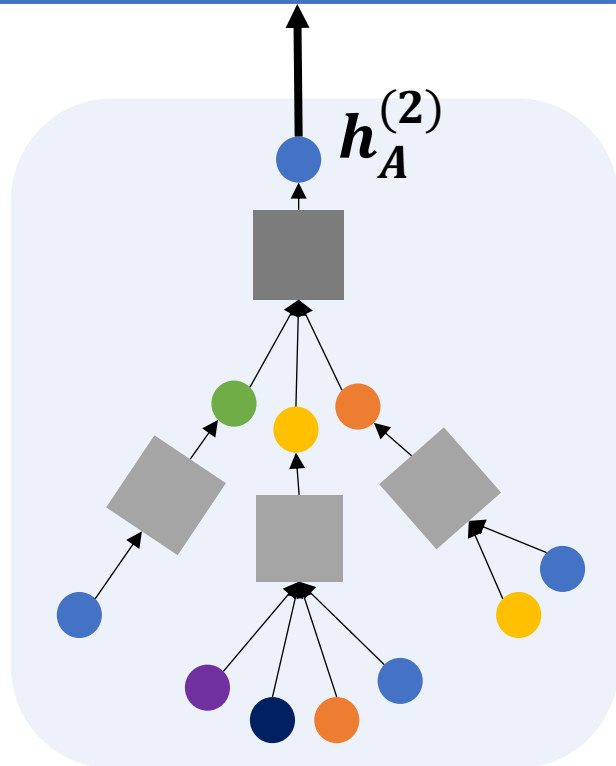


Node-level prediction tasks



Node-level prediction tasks

Node
classification



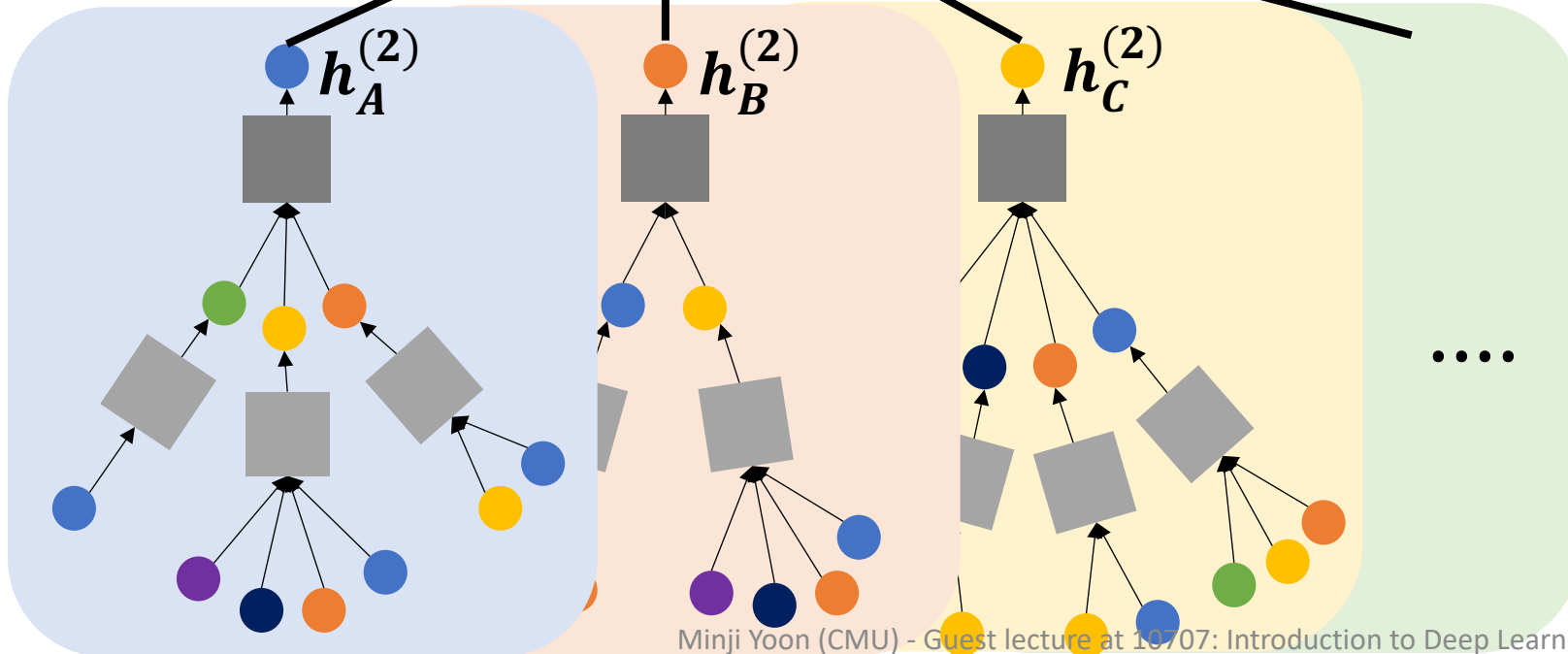
- Classify **papers** into topics on **citation networks**
- Cluster **posts** into subgroups on **Reddit networks**
- Classify **products** into categories on **Amazon co-purchase graphs**

Graph-level prediction tasks

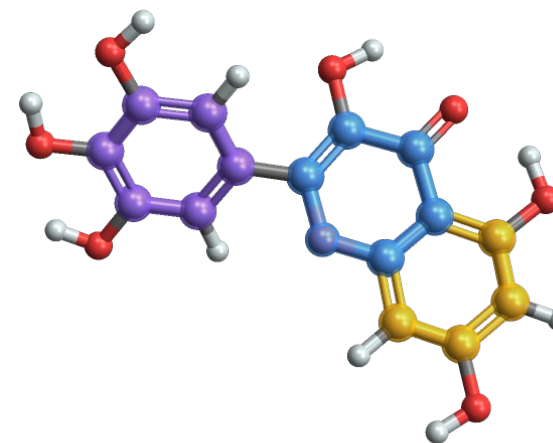
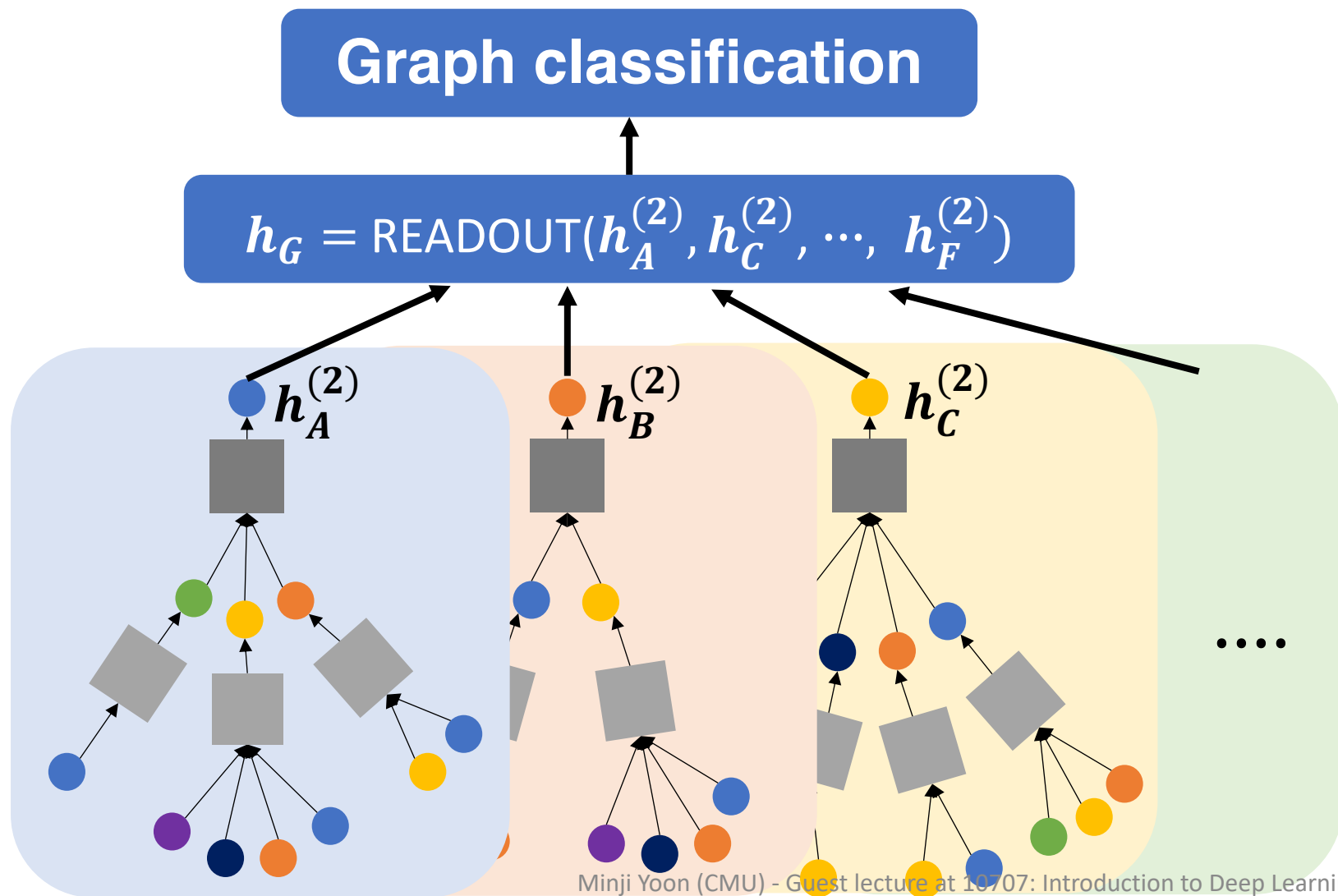
Graph classification

$$h_G = \text{READOUT}(h_A^{(2)}, h_C^{(2)}, \dots, h_F^{(2)})$$

(ex) sum, average, min/max pooling of node embeddings



Graph-level prediction tasks



- Predict **properties of a molecule (graph)** where nodes are atoms and edges are chemical bonds

So far, we have talked about..

1. Graph Neural Network

- Problem definition
- Skeleton
 - Aggregation operation
 - Transformation operation

2. Implementation

- Computation graph
- Batch execution

3. Downstream tasks

- Node-level prediction
- Graph-level prediction

So far, we have talked about..

1. Graph Neural Network

- Problem definition
- Skeleton
 - Aggregation operation
 - Transformation operation

2. Implementation

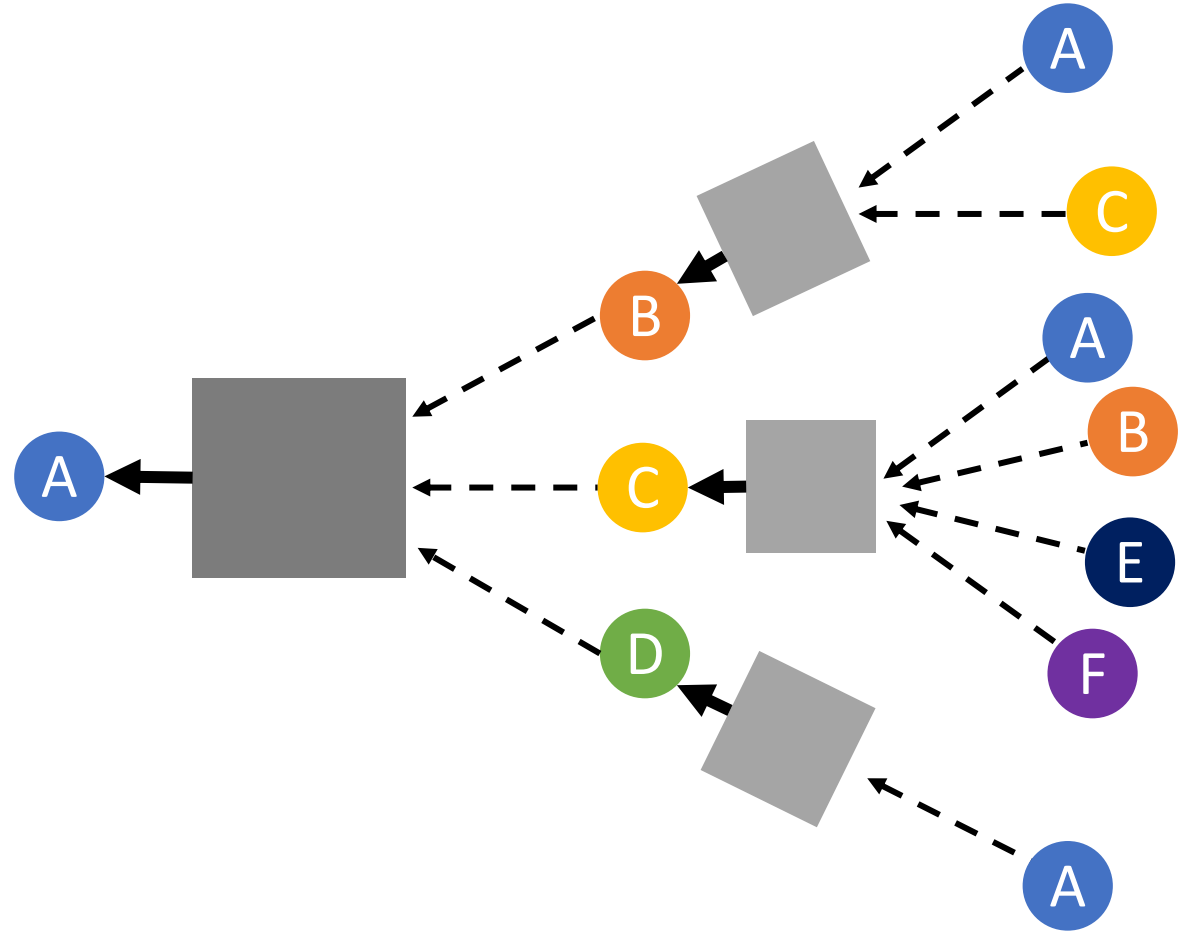
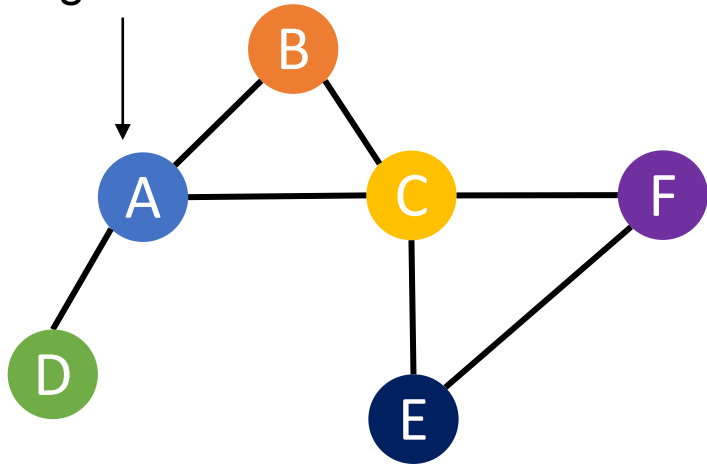
- Computation graph
- Batch execution

3. Downstream tasks

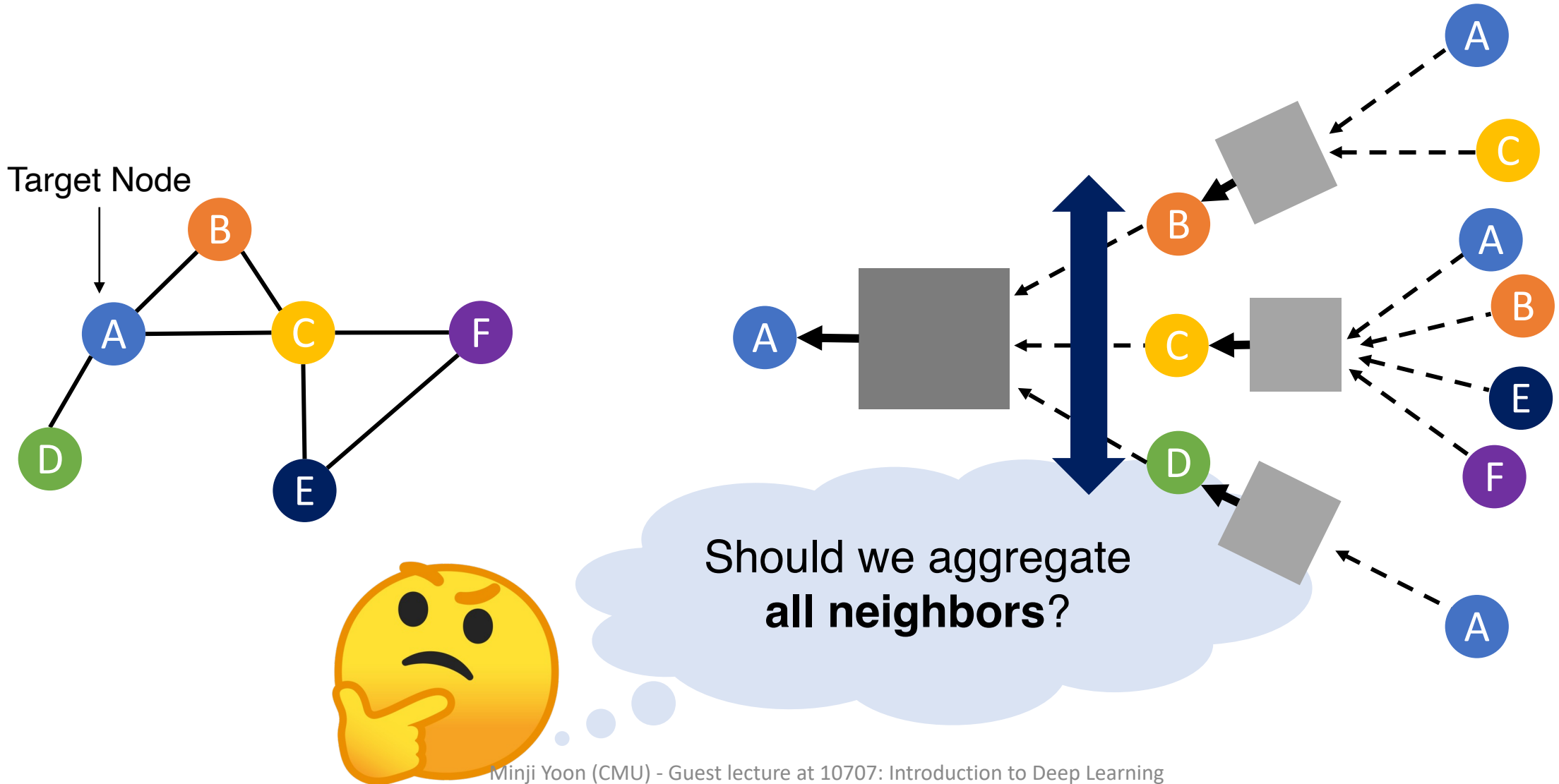
- Node-level prediction
- Graph-level prediction

Graph Neural Networks

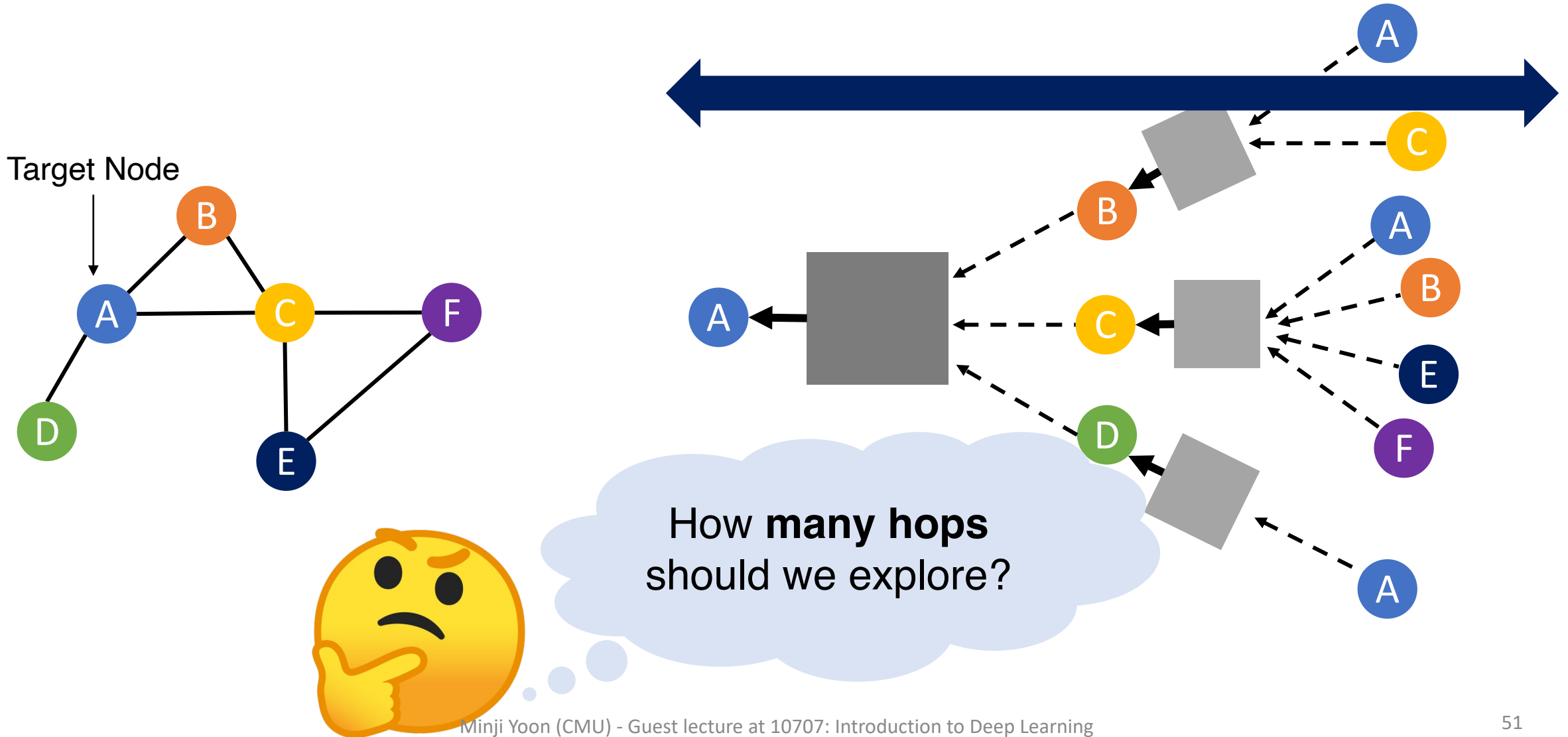
Target Node



Graph Neural Networks - Width

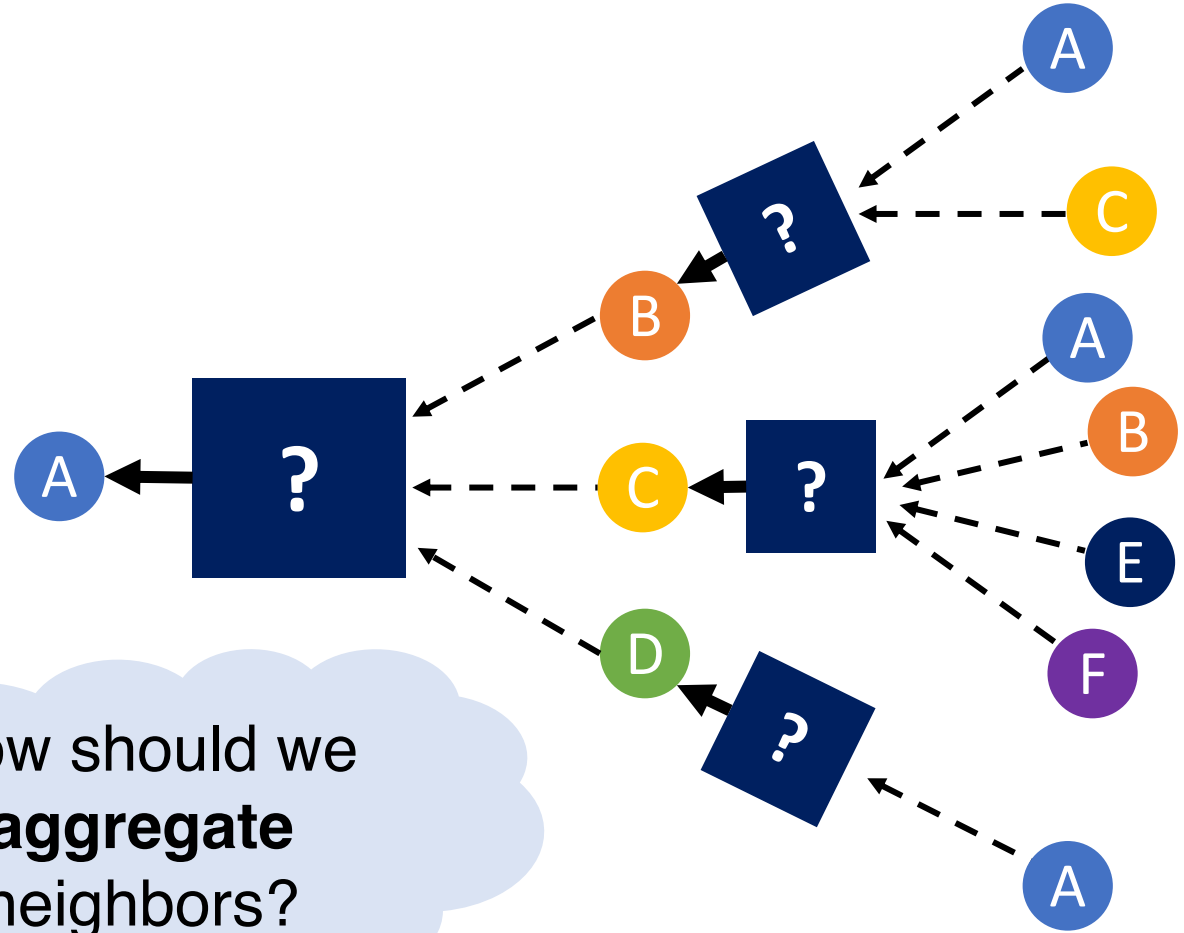
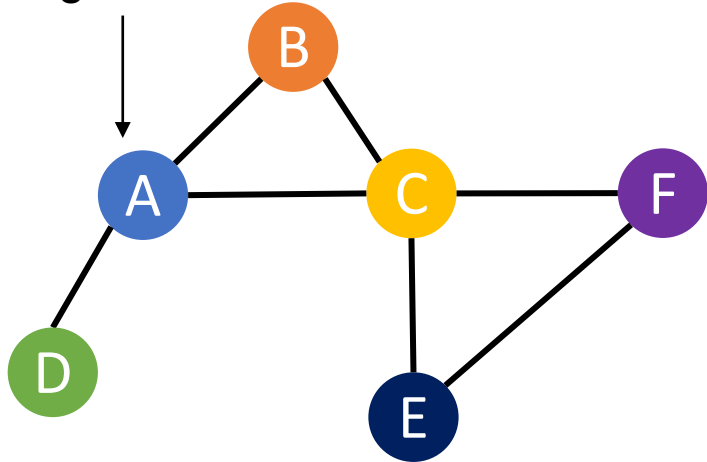


Graph Neural Networks - Depth



Graph Neural Networks - Aggregation

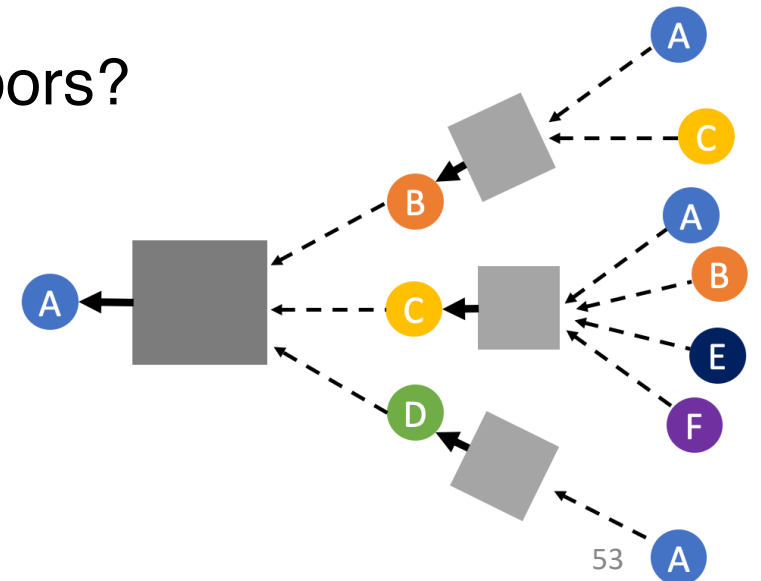
Target Node



How should we **aggregate** neighbors?

Graph Neural Network Architectures

- Width
 - Which neighbors should we aggregate messages from?
- Depth
 - How many hops should we check?
- Aggregation
 - How should we aggregate messages from neighbors?



Graph Neural Network Architectures

- **Width**

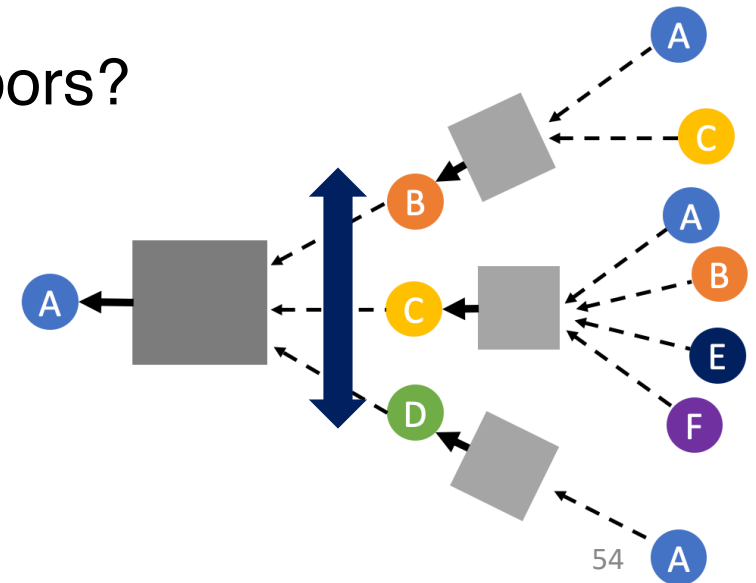
- Which neighbors should we aggregate messages from?

- **Depth**

- How many hops should we check?

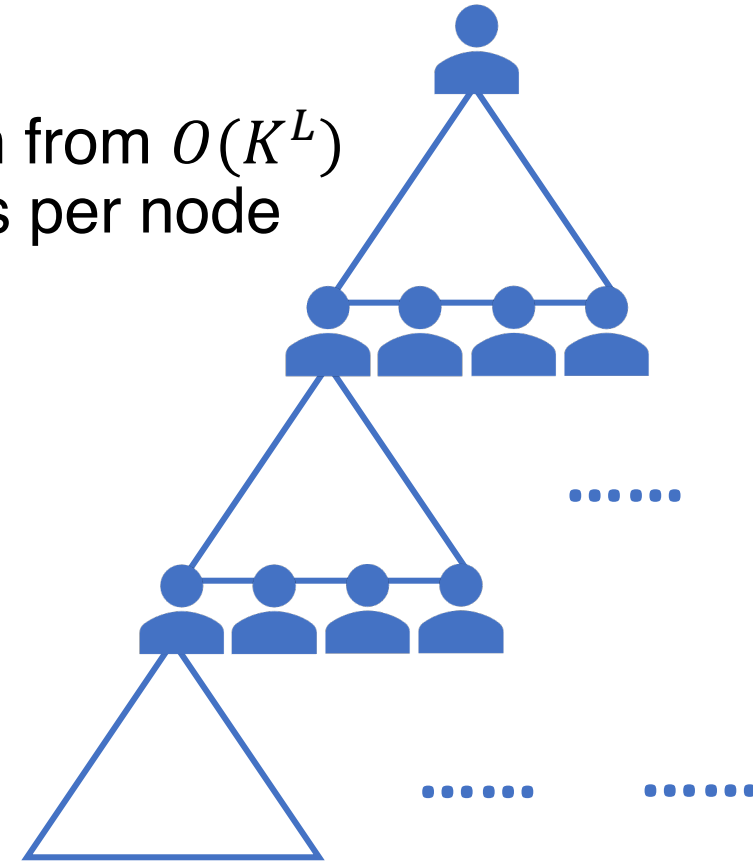
- **Aggregation**

- How should we aggregate messages from neighbors?



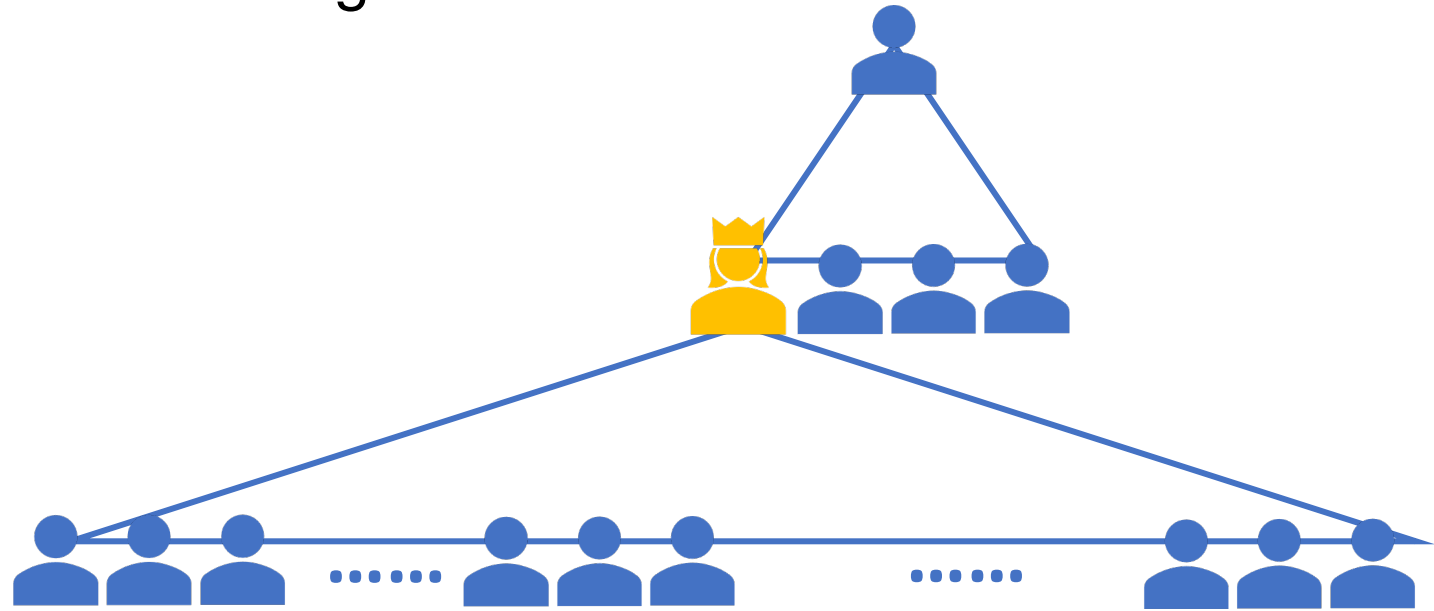
Aggregation Width in GNNs

- If we aggregate all neighbors, GNNs have scalability issues
- Neighbor explosion
 - In L -layer GNNs, one node aggregates information from $O(K^L)$ nodes where K is the average number of neighbors per node



Aggregation Width in GNNs

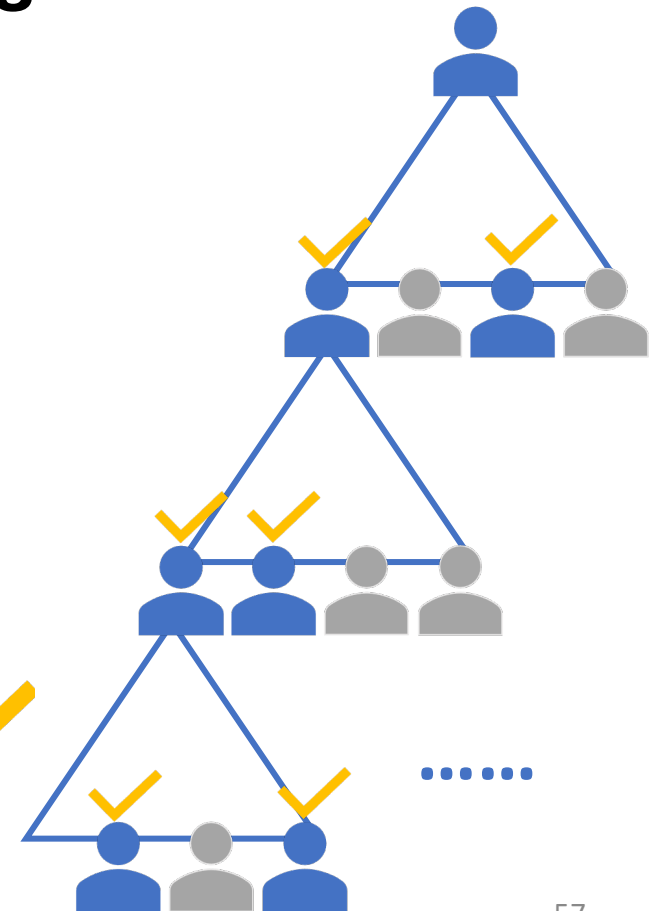
- If we aggregate all neighbors, GNNs have scalability issues
- Neighbor explosion
 - Hub nodes who are connected to a huge number of nodes



Aggregation Width in GNNs

- Limit the neighborhood expansion by **sampling** a fixed number of neighbors

Sample the neighbors ✓



Aggregation Width in GNNs

- Random sampling
 - Assign **same** sampling probabilities to all neighbors
 - *GraphSage*^[4]
- Importance sampling
 - Assign **different** sampling probabilities to all neighbors
 - *FastGCN*^[5], *LADIES*^[6], *AS-GCN*^[7], *GCN-BS*^[8], *PASS*^[9]

[4] Will Hamilton, et al. “Inductive representation learning on large graphs”

[5] Jie Chen, et al. “Fastgcn: fast learning with graph convolutional networks via importance sampling”

[6] Difan Zou, et al. “Layer-Dependent Importance Sampling for Training Deep and Large Graph Convolutional Networks”

[7] Wenbing Huang, et al. “Adaptive sampling towards fast graph representation learning”

[8] Ziqi Liu, et al. “Bandit Samplers for Training Graph Neural Networks”

[9] Minji Yoon, et al. “Performance-Adaptive Sampling Strategy Towards Fast and Accurate Graph Neural Networks”

Aggregation Width in GNNs

Importance sampling

: assign *higher sampling probabilities to neighbors who*

- **Minimize variance in sampling**
 - *FastGCM*^[5], *LADIES*^[6], *AS-GCM*^[7], *GCN-BS*^[8]
- **Maximize GNN performance**
 - *PASS*^[9]

[4] Will Hamilton, et al. “Inductive representation learning on large graphs”

[5] Jie Chen, et al. “Fastgcn: fast learning with graph convolutional networks via importance sampling”

[6] Difan Zou, et al. “Layer-Dependent Importance Sampling for Training Deep and Large Graph Convolutional Networks”

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Aggregation Width in GNNs

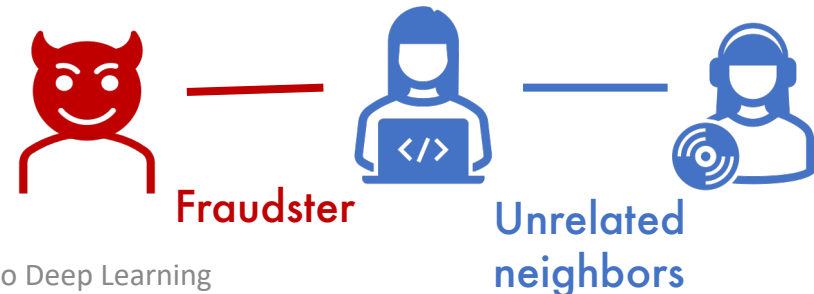
Method	Cora	Citeseer	Pubmed	AmazonC	AmazonP	MsCS	MsPhysics
FastGCN	0.582	0.496	0.569	0.480	0.542	0.520	0.638
AS-GCN	0.462	0.387	0.502	0.419	0.480	0.403	0.516
GraphSage	0.788	0.698	0.792	0.707	0.787	0.766	0.875
GCN-BS	0.788	0.693	0.809	0.736	0.800	0.780	0.887
PASS	0.821	0.715	0.858	0.757	0.855	0.884	0.934

- Node classification task on 7 different real-world graphs
- PASS outperforms all variance-minimizing methods by up to 10.4%

Aggregation Width in GNNs

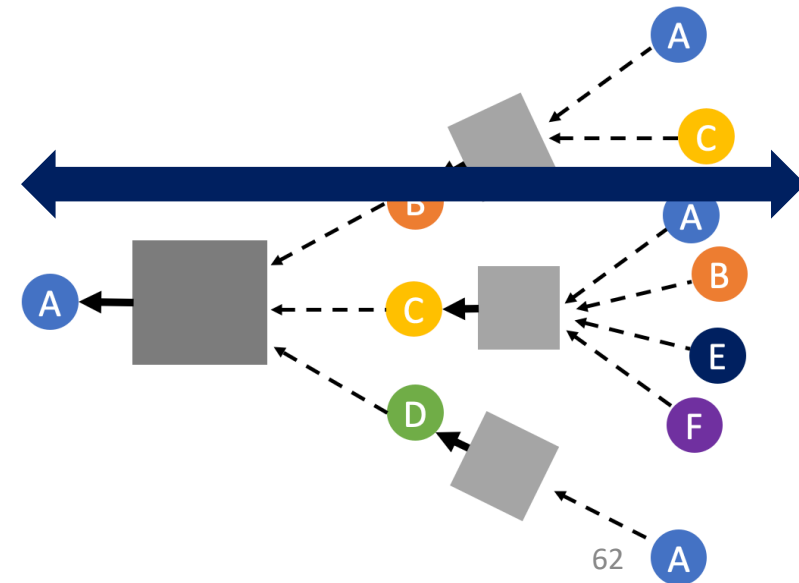
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Real-world graphs are noisy!!



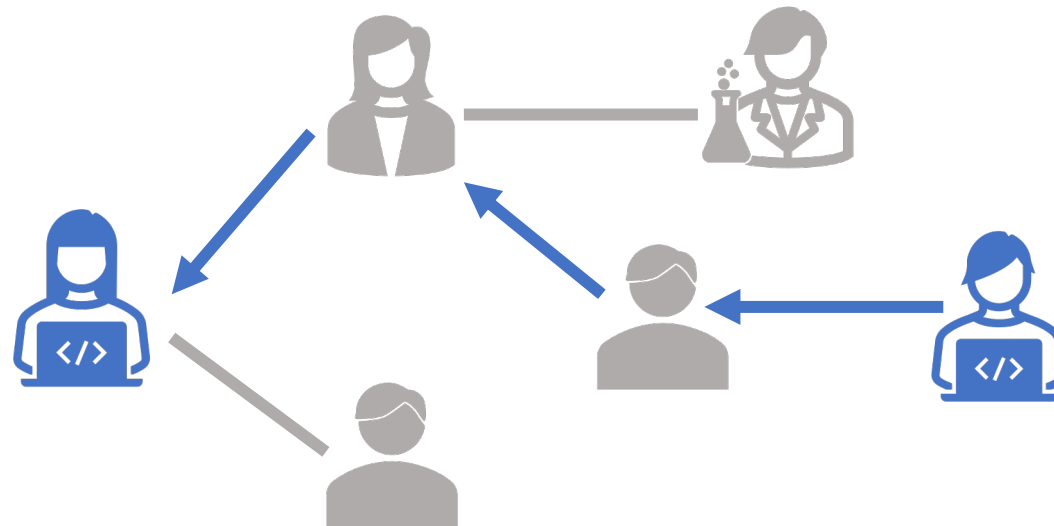
Graph Neural Network Architectures

- Width
 - Which neighbors should we aggregate messages from?
- **Depth**
 - **How many hops should we check?**
- Aggregation
 - How should we aggregate messages from neighbors?



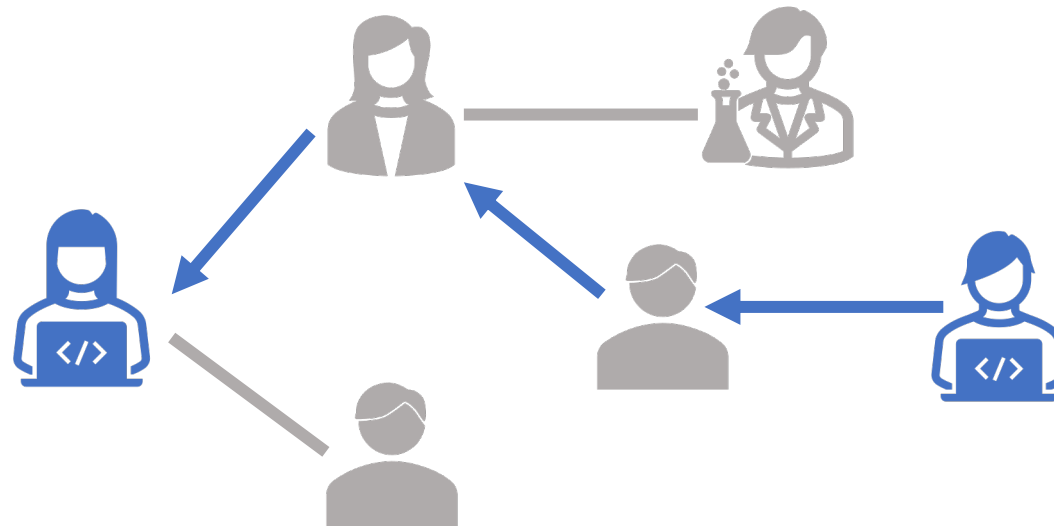
Aggregation Depth in GNNs

- Informative neighbors could be indirectly connected with a target node



Aggregation Depth in GNNs

- Informative neighbors could be indirectly connected with a target node
- Can't we just look multiple hops away from the target node?



Aggregation Depth in GNNs

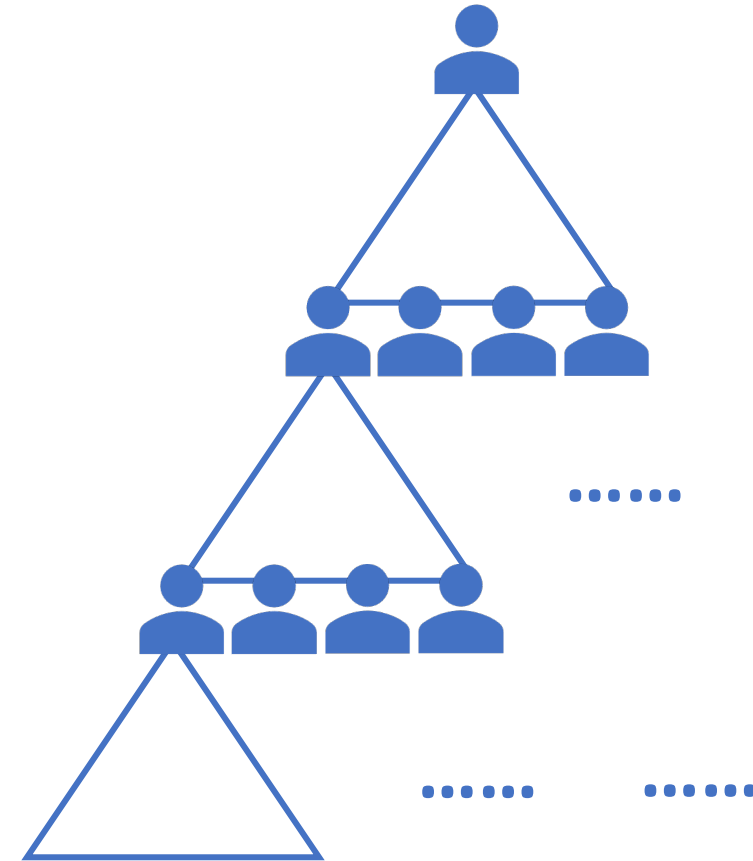
- 2-layer or 3-layer GNNs are commonly used in real worlds



Wasn't it Deeeep Learning?

Aggregation Depth in GNNs

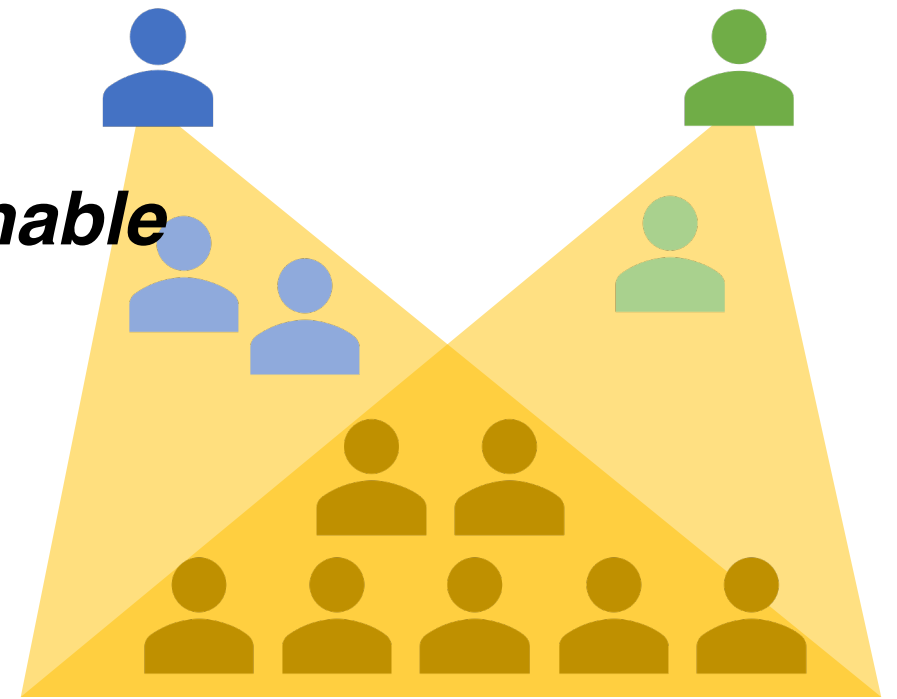
- When we increase the depth L more than this, GNNs face neighbor explosion $O(K^L)$
 - **Over-smoothing**
 - **Over-squashing**



Aggregation Depth in GNNs

Over-smoothing^[10]

- When GNNs become deep, nodes share many neighbors
- Node embeddings become *indistinguishable*

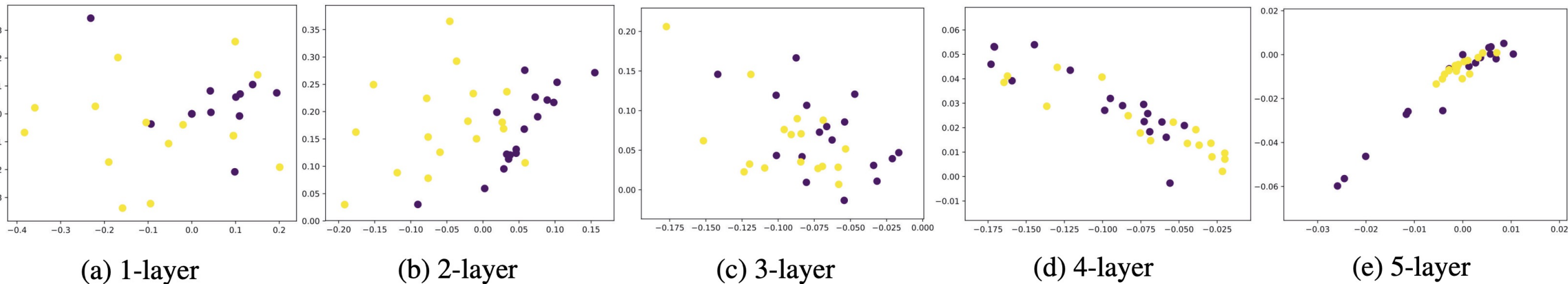


[10] Qimai Li, et al. "Deeper Insights into Graph Convolutional Networks for Semi-Supervised Learning"

Aggregation Depth in GNNs

Over-smoothing^[10]

- Node embeddings of Zachary's karate club network with GNNs

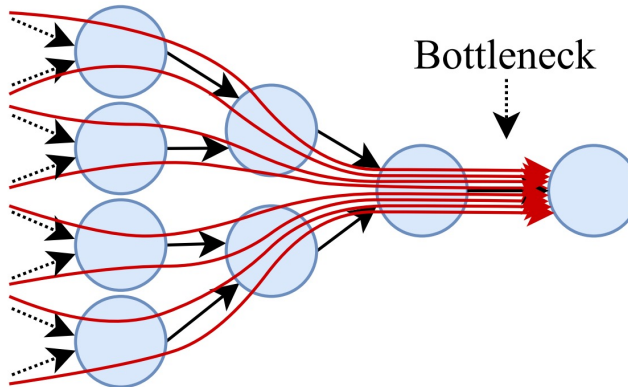


[10] Qimai Li, et al. "Deeper Insights into Graph Convolutional Networks for Semi-Supervised Learning"

Aggregation Depth in GNNs

Over-squashing^[12]

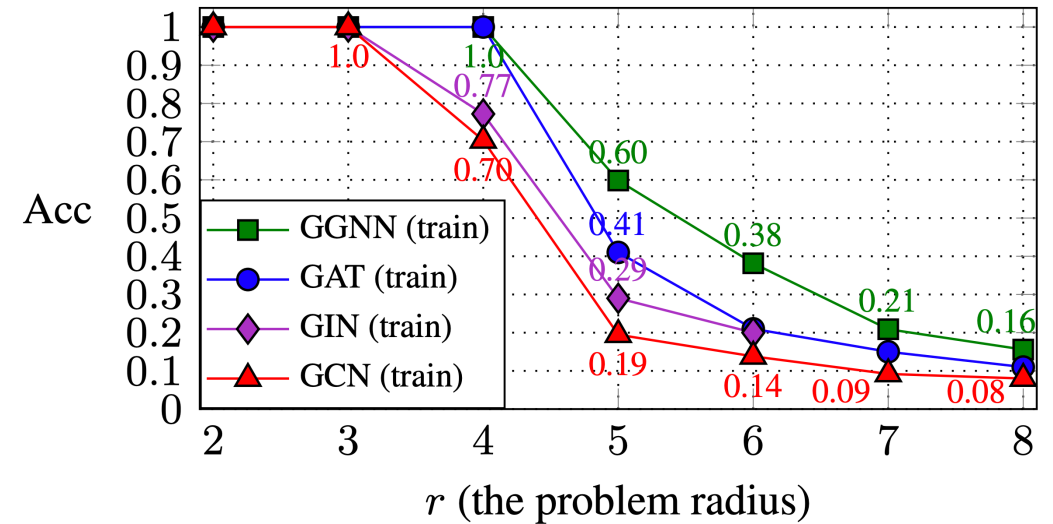
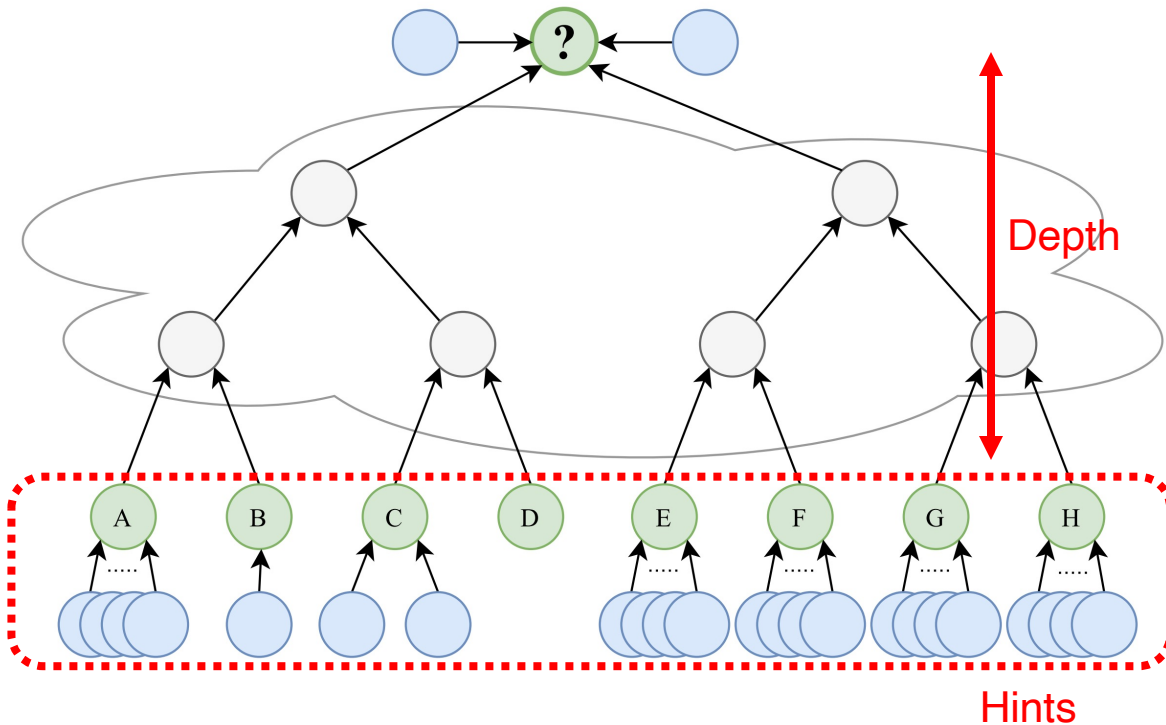
- A node's exponentially-growing neighborhood is compressed into a fixed-size vector



[12] Uri Alon, et al. "ON THE BOTTLENECK OF GRAPH NEURAL NETWORKS AND ITS PRACTICAL IMPLICATIONS"

Aggregation Depth in GNNs

Over-squashing^[12]



[12] Uri Alon, et al. "ON THE BOTTLENECK OF GRAPH NEURAL NETWORKS AND ITS PRACTICAL IMPLICATIONS"

Aggregation Depth in GNNs

Decoupling the two concepts of depths in GNNs^[13]

- **Depth-1**: neighborhood that each node aggregates information from
- **Depth-2**: number of layers in GNNs

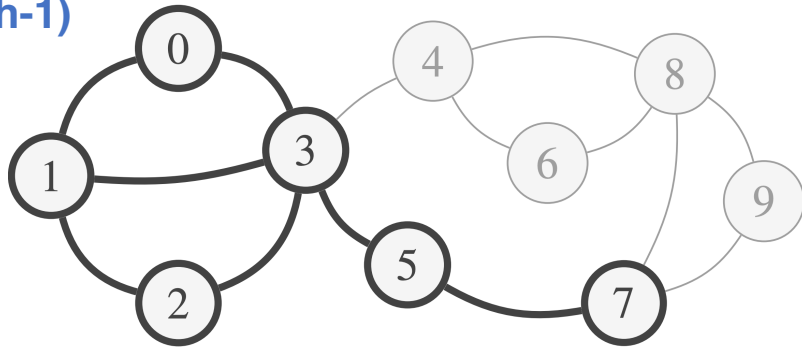
[13] Hanqing Zeng, et al. "Decoupling the Depth and Scope of Graph Neural Networks"

Aggregation Depth in GNNs

Decoupling the two concepts of depths in GNNs^[13]

- **Depth-1**: neighborhood that each node aggregates information from
- **Depth-2**: number of layers in GNNs

Depth of neighborhood
(Depth-1)



$$\mathcal{G}_s = \text{SAMPLE}(\mathcal{G})$$

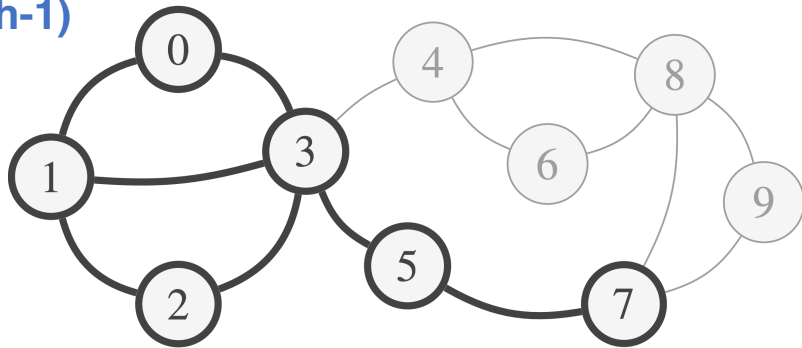
[13] Hanqing Zeng, et al. "Decoupling the Depth and Scope of Graph Neural Networks"

Aggregation Depth in GNNs

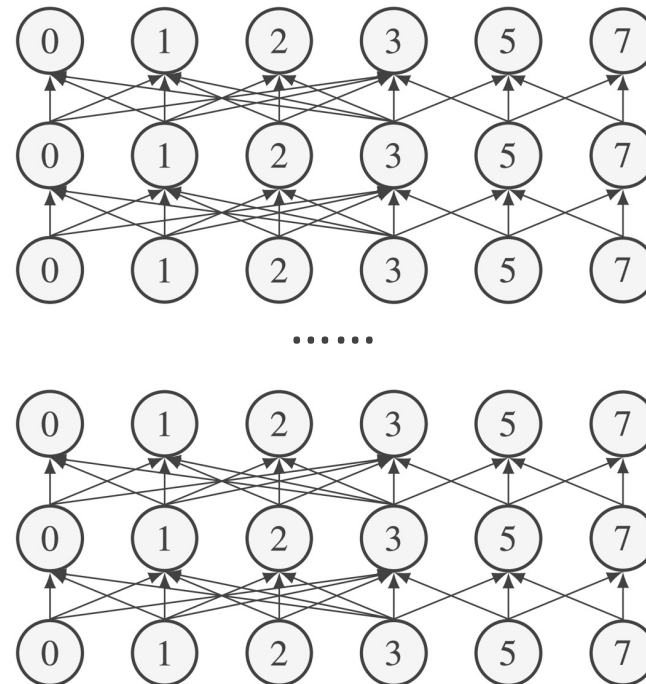
Decoupling the two concepts of depths in GNNs^[13]

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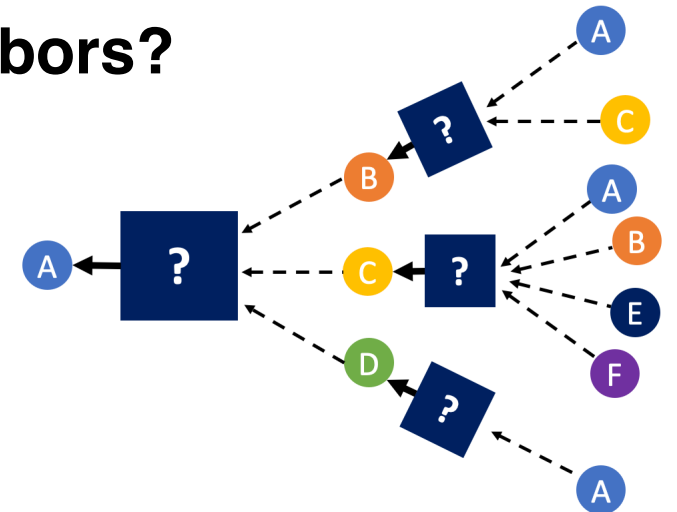


Depth of GNN
(Depth-2)

[13] Hanqing Zeng, et al. "Decoupling the Depth and Scope of Graph Neural Networks"

Graph Neural Network Architectures

- Width
 - Which neighbors should we aggregate messages from?
- Depth
 - How many hops should we check?
- **Aggregation**
 - **How should we aggregate messages from neighbors?**



Aggregation strategy in GNNs

In each layer l :

Aggregate over neighbors

$$m_v^{(l-1)} = \mathbf{f}^{(l)}\left(h_v^{(l-1)}, \{h_u^{(l-1)} : u \in \mathcal{N}(v)\}\right)$$

Transform messages

$$h_v^{(l)} = \mathbf{g}^{(l)}(m_v^{(l-1)})$$

Aggregation strategy in GNNs

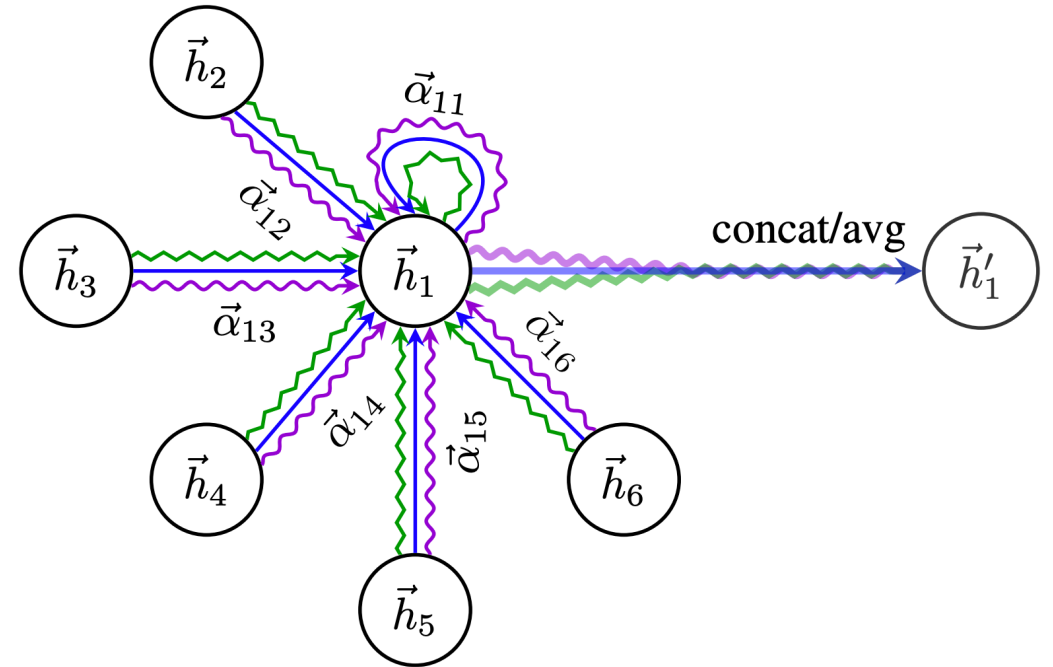
- GCN^[1]
 - Average embeddings of neighboring nodes

[1] Kipf, Thomas N., et al. "Semi-supervised classification with graph convolutional networks."

Aggregation strategy in GNNs

- GAT^[14]
 - Different weights to different nodes in a neighborhood
 - Multi-head attention

$$\alpha_{ij} = \frac{\exp\left(\text{LeakyReLU}\left(\vec{\mathbf{a}}^T [\mathbf{W}\vec{h}_i \parallel \mathbf{W}\vec{h}_j]\right)\right)}{\sum_{k \in \mathcal{N}_i} \exp\left(\text{LeakyReLU}\left(\vec{\mathbf{a}}^T [\mathbf{W}\vec{h}_i \parallel \mathbf{W}\vec{h}_k]\right)\right)}$$



[14] Petar Veličković, et al. "GRAPH ATTENTION NETWORKS."

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In each layer l :

Aggregate over neighbors

$$m_v^{(l-1)} = \mathbf{f}^{(l)}\left(h_v^{(l-1)}, \{h_u^{(l-1)} : u \in \mathcal{N}(v)\}\right)$$

Core part of GNNs

Transform messages

$$h_v^{(l)} = \mathbf{g}^{(l)}(m_v^{(l-1)})$$

Any neural network module can fit in.
1-layer MLP is commonly used.

Aggregation strategy in GNNs

Power of **GNNs**

=

Power of **aggregation strategies**

Aggregation strategy in GNNs

- By measuring the power of GNNs, we can find the best aggregation strategy!!



Aggregation strategy in GNNs

- By measuring the expressive power of GNNs, we can find the best aggregation strategy!!
- *But.. what is the power of GNNs and how can we measure it?*



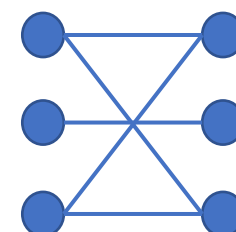
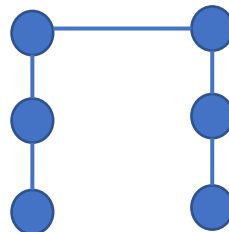
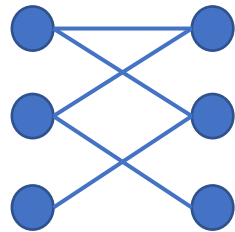
Aggregation strategy in GNNs

- How powerful are Graph Neural Networks?^[2]
- Metric
 - Graph-level prediction task
 - Can a GNN model distinguish two non-isomorphic graphs?

[2] Keyulu Xu., et al. "HOW POWERFUL ARE GRAPH NEURAL NETWORKS?"

Aggregation strategy in GNNs

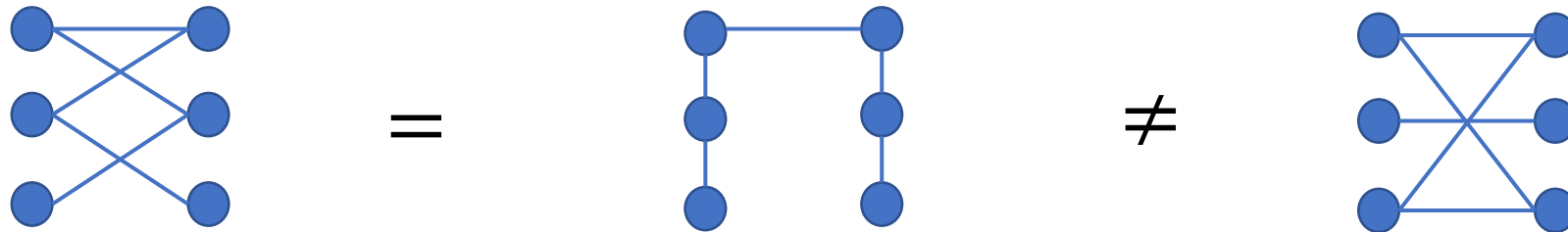
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[2] Keyulu Xu., et al. "HOW POWERFUL ARE GRAPH NEURAL NETWORKS?"

Aggregation strategy in GNNs

- How powerful are Graph Neural Networks?^[2]
 - Any aggregation-based GNN is at most as powerful as the **WL test**^[15]
 - Maximum power = aggregation strategy is injective

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$


[2] Keyulu Xu., et al. "HOW POWERFUL ARE GRAPH NEURAL NETWORKS?"

[15] Boris Weisfeiler and AA Leman. "A reduction of a graph to a canonical form and an algebra arising during this reduction"

Aggregation strategy in GNNs

- How powerful are Graph Neural Networks?^[2]
 - Any aggregation-based GNN is at most as powerful as the **WL test**^[15]
 - Maximum power = aggregation strategy is injective
 - (ex) summation



Mean and Max both fail, while Sum can distinguish them!!

[2] Keyulu Xu., et al. "HOW POWERFUL ARE GRAPH NEURAL NETWORKS?"

[15] Boris Weisfeiler and AA Leman. "A reduction of a graph to a canonical form and an algebra arising during this reduction"

Aggregation strategy in GNNs

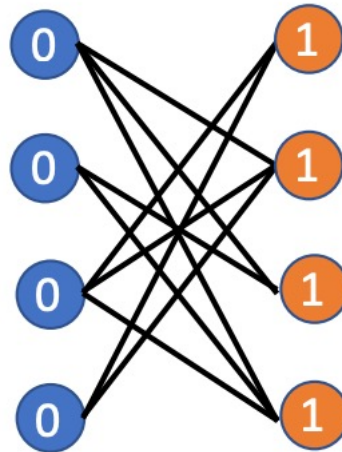
- Can we make more powerful GNNs?
 - Very active area, with many open problems

Aggregation strategy in GNNs

- Homophily assumption
 - Connected nodes are similar/related/informative

Aggregation strategy in GNNs

- Homophily assumption
 - Connected nodes are similar/related/informative
- How can we deal with **heterophilous networks**?^[21,22]
 - Connected nodes have different class labels and dissimilar features



[21] Jiong Zhu., et al. "Beyond Homophily in Graph Neural Networks: Current Limitations and Effective Designs"

[22] Yao Ma, et al. "IS HOMOPHILY A NECESSITY FOR GRAPH NEURAL NETWORKS?"

Aggregation strategy in GNNs

Improved accuracy
after filtering datasets

- Heterophilous graph datasets have serious drawbacks^[23]



	accuracy on original dataset	squirrel accuracy on filtered dataset	ranks
ResNet	33.88 ± 1.79	36.55 ± 1.82	2 / 7
ResNet+SGC	34.36 ± 1.21	38.36 ± 1.97	11 / 1
ResNet+adj	65.46 ± 1.58	38.37 ± 1.99	2 / 1
GCN	39.06 ± 1.52	39.47 ± 1.47	6 / 2
SAGE	35.83 ± 1.32	36.09 ± 1.99	9 / 9
GAT	32.21 ± 1.63	35.62 ± 2.06	14 / 11
GAT-sep	35.72 ± 1.98	35.46 ± 3.10	10 / 13
GT	31.61 ± 1.10	36.30 ± 1.98	15 / 8
GT-sep	36.08 ± 1.58	36.66 ± 1.63	8 / 6
H ₂ GCN	29.45 ± 1.65	35.10 ± 1.15	17 / 15
CPGNN	30.91 ± 1.98	30.04 ± 2.03	16 / 16
GPR-GNN	33.39 ± 2.05	38.95 ± 1.99	13 / 3
FSGNN	68.93 ± 1.69	35.92 ± 1.32	1 / 10
GloGNN	61.21 ± 1.96	35.11 ± 1.24	3 / 14
FAGCN	47.63 ± 1.85	41.08 ± 2.27	4 / 1
GBK-GNN	37.06 ± 1.24	35.51 ± 1.65	7 / 12
JacobiConv	46.17 ± 4.34	29.71 ± 1.66	5 / 17

[23] Oleg Platonov., et al. "A critical look at the evaluation of GNNs under heterophily: are we really making progress?"

So far, we have talked about..

1. Graph Neural Network

- Problem definition
- Skeleton: aggregation, transformation operations

2. Research questions in GNN architectures

- Width
- Depth
- Aggregation

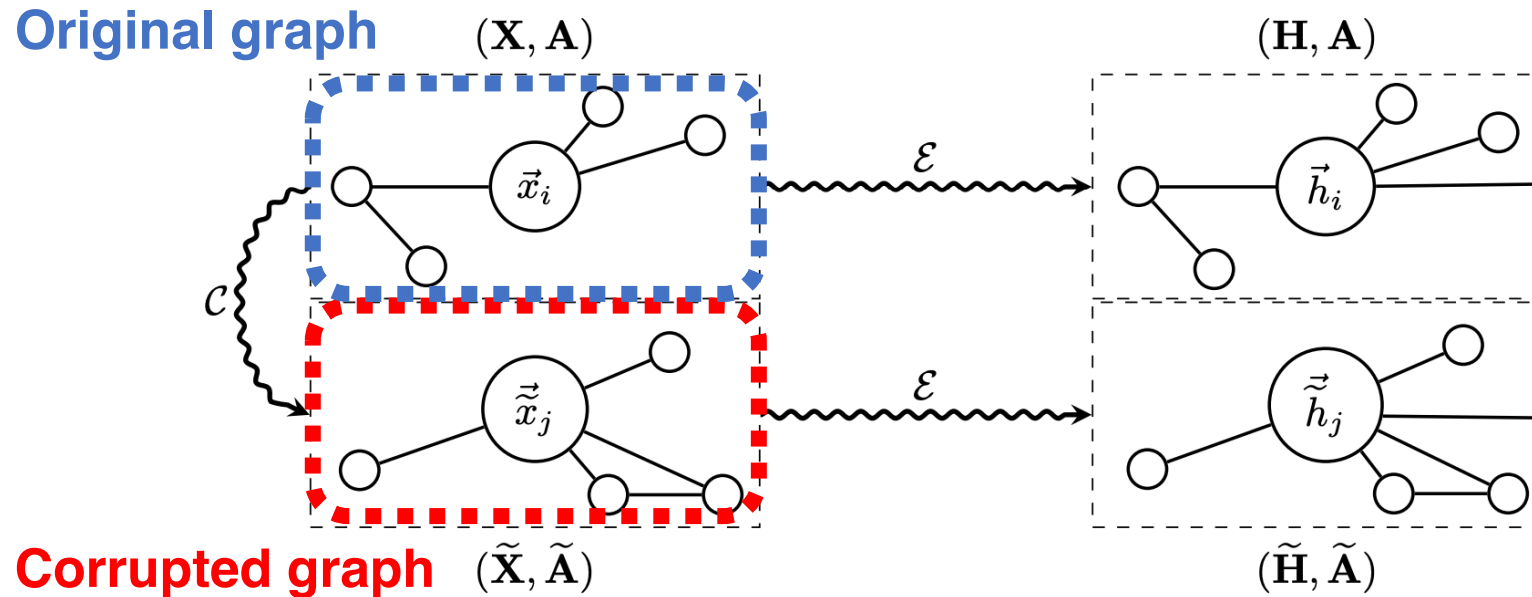
3. GNN training strategy

How to train GNNs

- Semi-supervised learning
 - Input node features are given for all nodes in a graph
 - Only a subset of nodes have labels

How to train GNNs

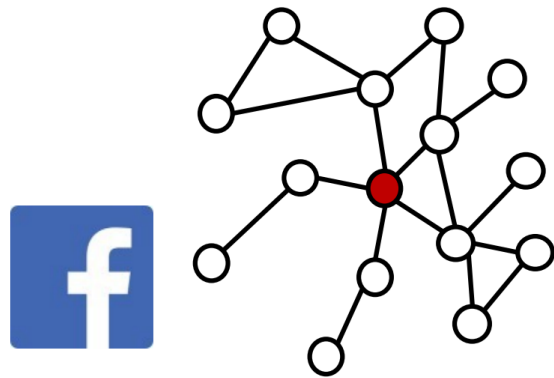
- Unsupervised learning^[26]
 - Contrastive learning



[26] Petar Veličković, et al. "DEEP GRAPH INFOMAX"

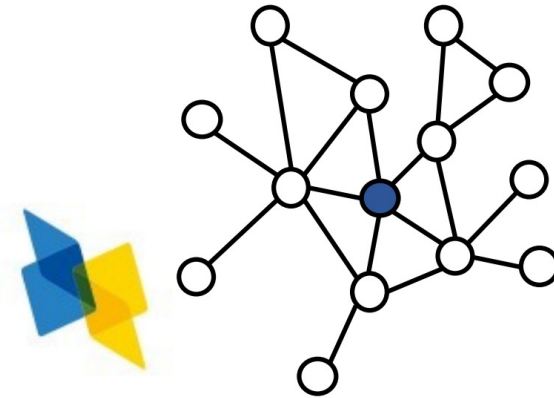
How to train GNNs

- Transfer learning
 - Transfer a pre-trained GNN model between graphs^[27]



Facebook network

Pre-trained GNN f

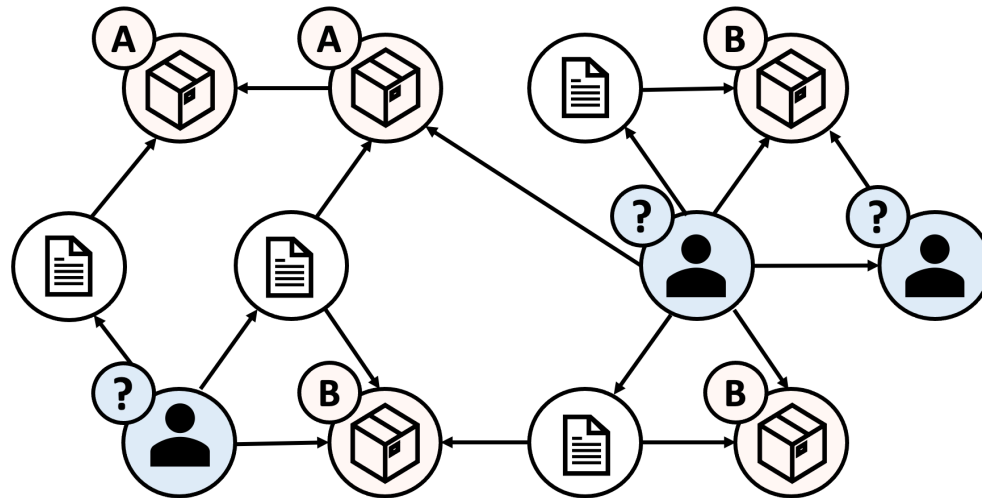


DBLP co-authorship network

[27] Jiezhong Qiu, et al. "GCC: Graph Contrastive Coding for Graph Neural Network Pre-Training"

How to train GNNs

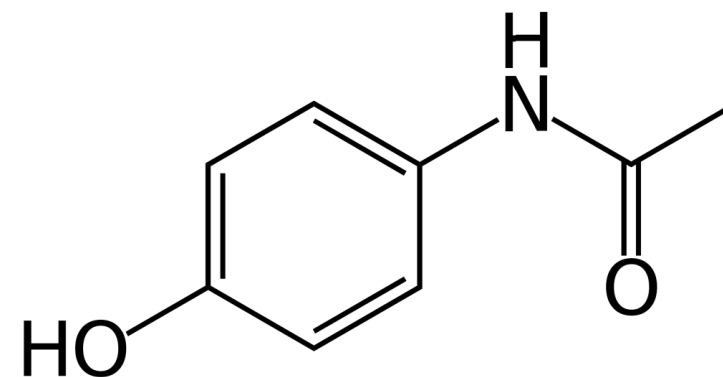
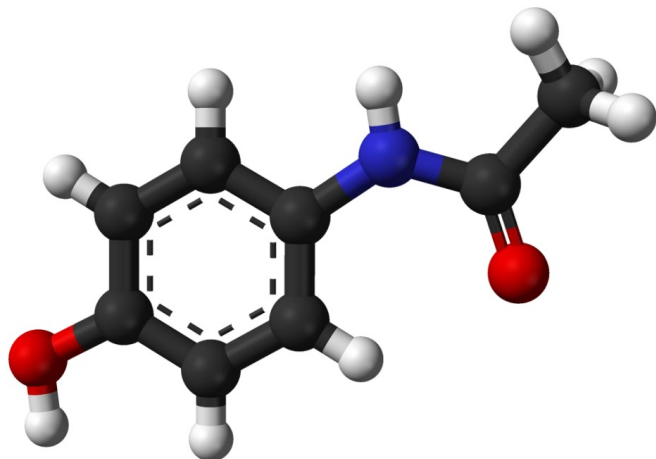
- Transfer learning
 - Transfer between different node types across a **heterogeneous graph**^[28]



[28] Minji Yoon, et al. "Zero-shot Domain Adaptation of Heterogeneous Graphs via Knowledge Transfer Networks "

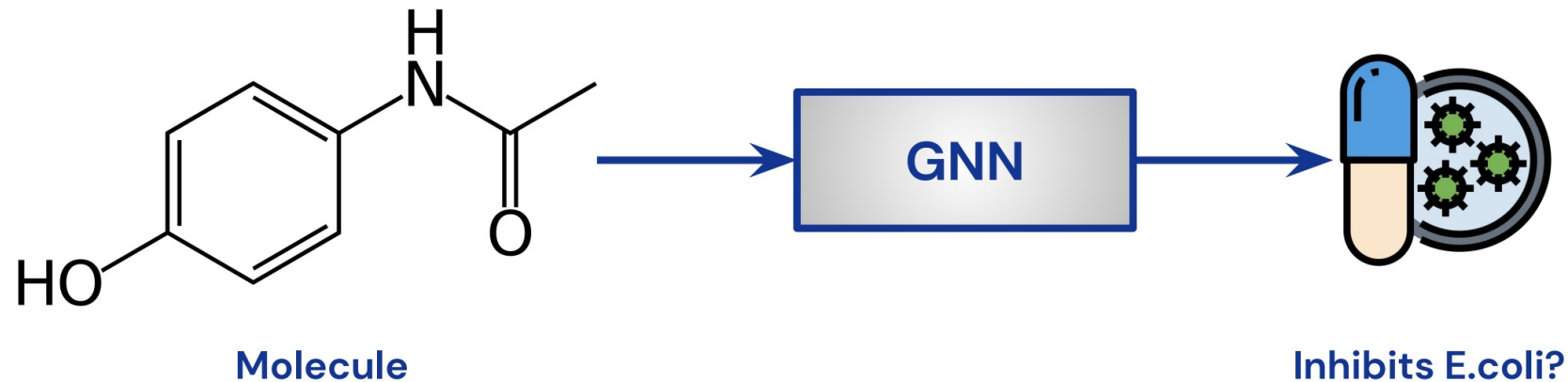
Impactful applications in science

- GNNs for molecule classification
- Molecule
 - Node: atoms
 - Edge: bonds
 - Input features: atom type, charge, bond type



Impactful applications in science

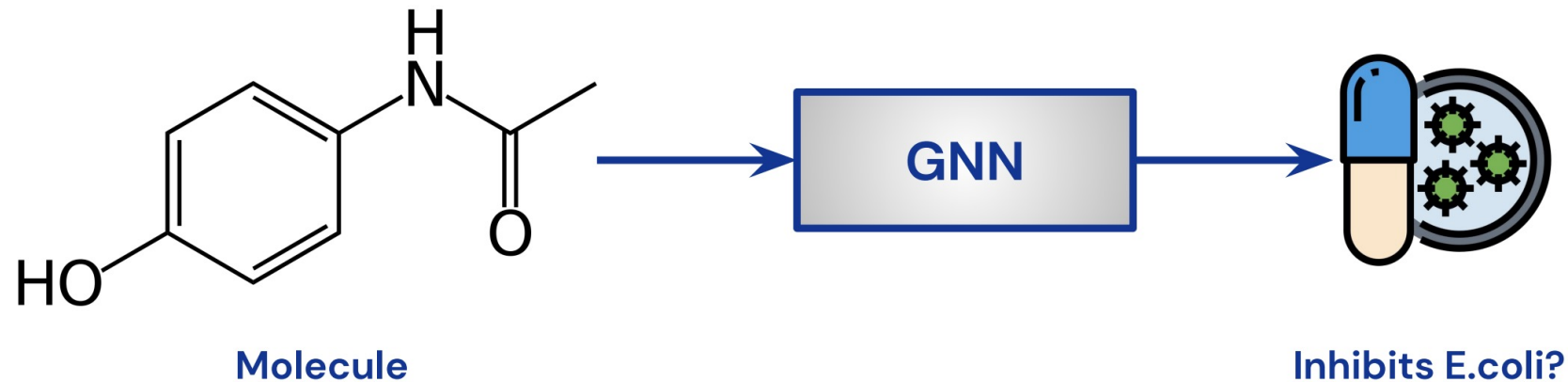
- Graph-level prediction: whether the molecule is a potent **drug**^[29]
 - Binary classification on whether the drug will inhibit certain bacteria



[29] Jonathan M. Stokes, et al. "A Deep Learning Approach to Antibiotic Discovery"

Impactful applications in science

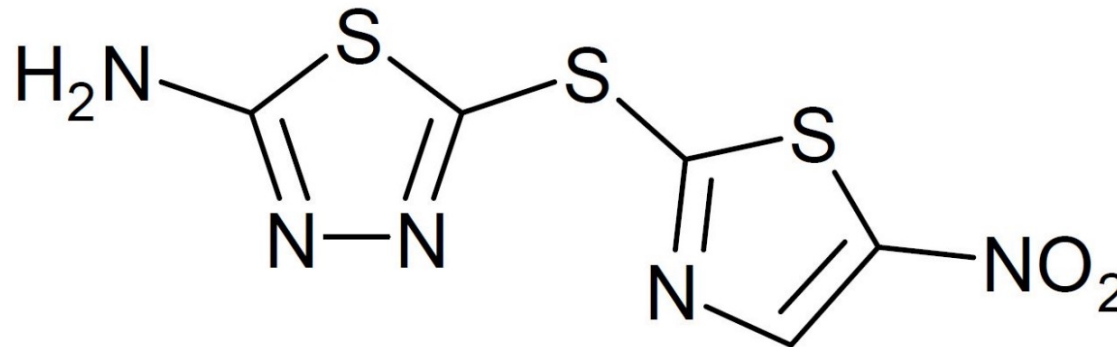
- Graph-level prediction: whether the molecule is a potent **drug**^[29]
 - Execute on a large dataset of known candidate molecules
 - Select the *~ top-100* candidates from the GNN model
 - Have chemists thoroughly investigate those



[29] Jonathan M.Stokes, et al. "A Deep Learning Approach to Antibiotic Discovery"

Impactful applications in science

- Discover a previously overlooked compound that is a **highly potent** antibiotic^[29]



Halicin

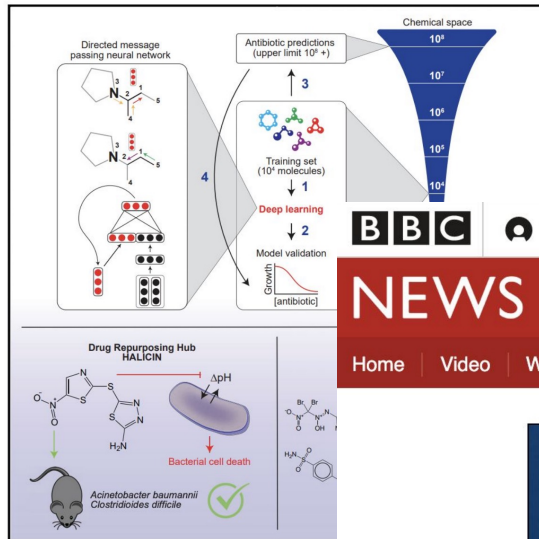
[29] Jonathan M. Stokes, et al. "A Deep Learning Approach to Antibiotic Discovery"

Impactful applications in science

Cell

A Deep Learning Approach to Antibiotic Discovery

Graphical Abstract



Authors

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So far, we have talked about..

1. Graph Neural Network
2. Research questions in GNN architectures
3. GNN training strategy
4. Applications in science

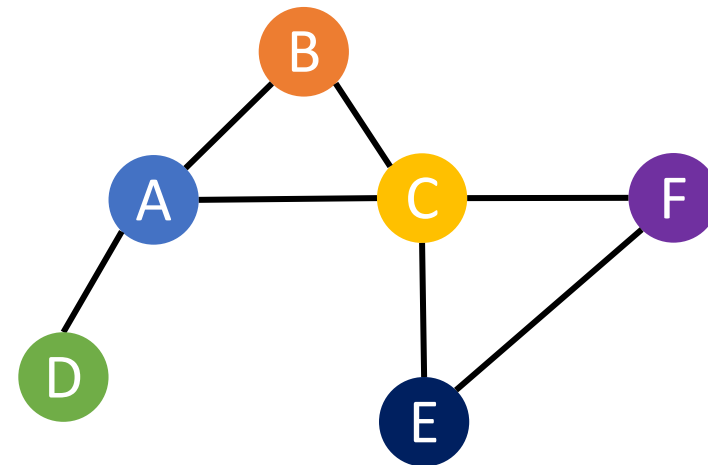
So far, we have talked about..

1. Graph Neural Network
2. Research questions in GNN architectures
3. GNN training strategy
4. Applications in science
- 5. Recent Directions inspired by other disciplines**
 - How to apply Transformers to graphs
 - How to utilize Large Language Models (LLMs) for graph tasks

Advanced

Graph Transformer

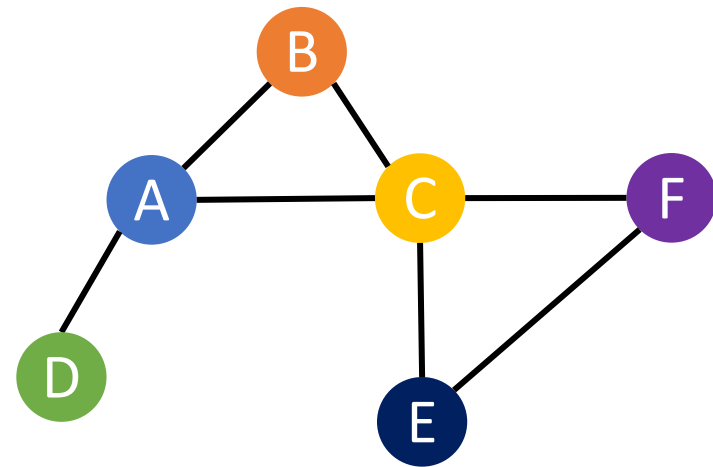
- Limitations of GNN
- Hard-coded structural inductive bias
 - Limited expressiveness (WL-test)
 - Oversmoothing
 - Oversquashing



Graph Transformer

- Aggregate from all nodes in a graph regardless of their connectivity

... Are we going to throw away the graph information?



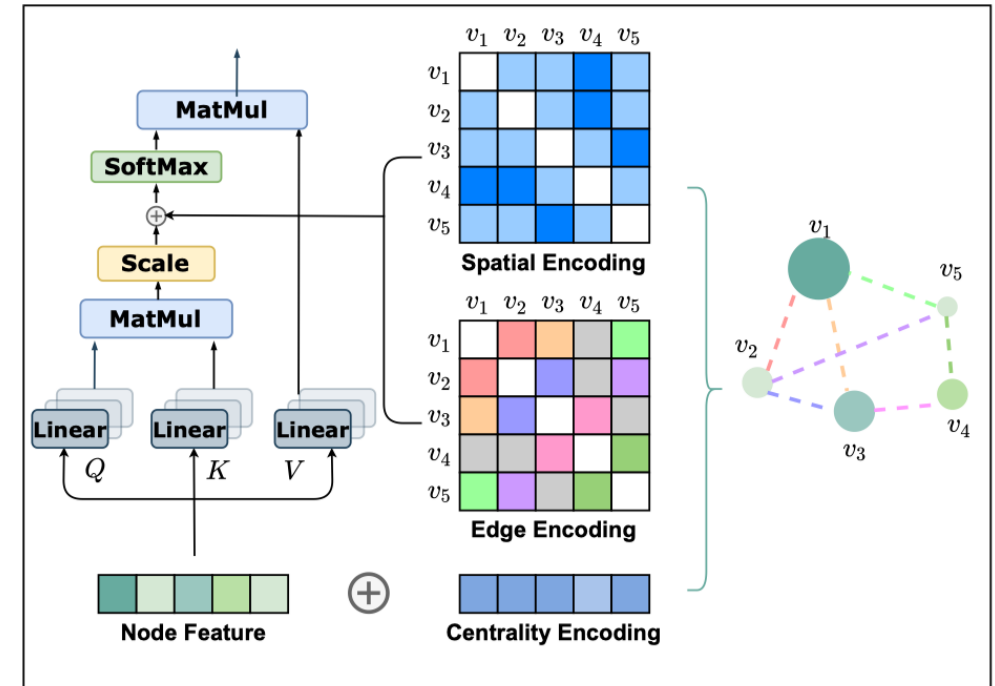
Graph Transformer

- Graphormer^[27]

$$h_i^{(0)} = x_i + z_{\text{deg}^-(v_i)}^- + z_{\text{deg}^+(v_i)}^+$$

$$A_{ij} = \frac{(h_i W_Q)(h_j W_K)^T}{\sqrt{d}} + b_{\phi(v_i, v_j)} + c_{ij}, \text{ where } c_{ij} = \frac{1}{N} \sum_{n=1}^N x_{e_n} (w_n^E)^T$$

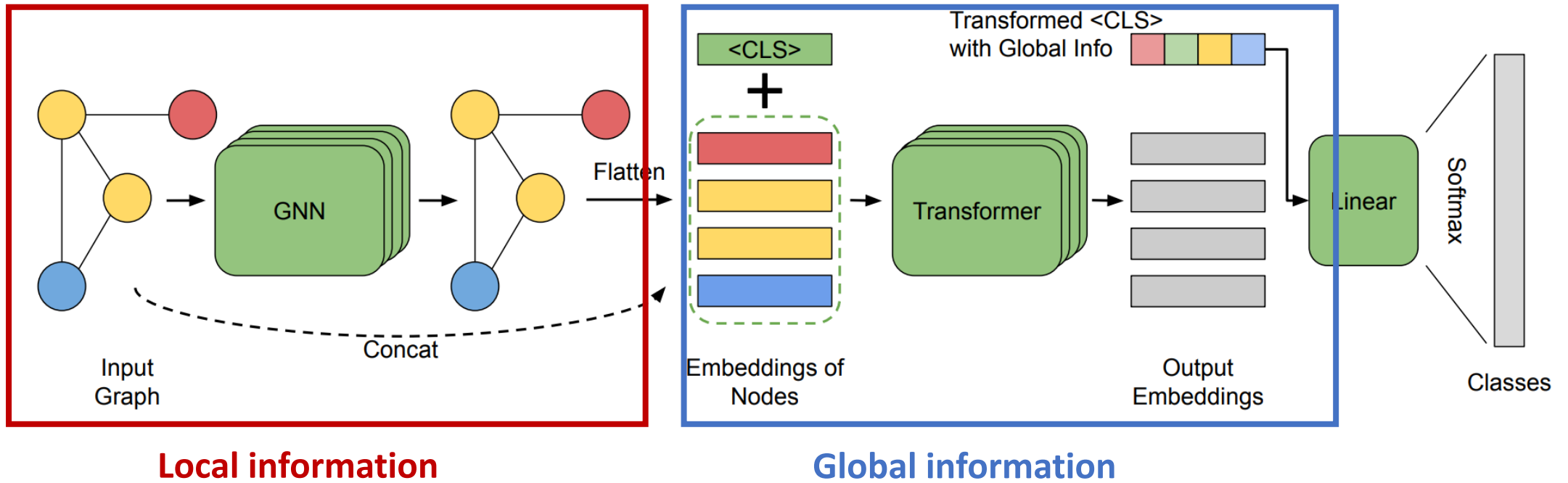
Degree information
Spatial distance information
Edge attribute information



[27] Chengxuan Ying, et al. "Do Transformers Really Perform Bad for Graph Representation?"

Graph Transformer

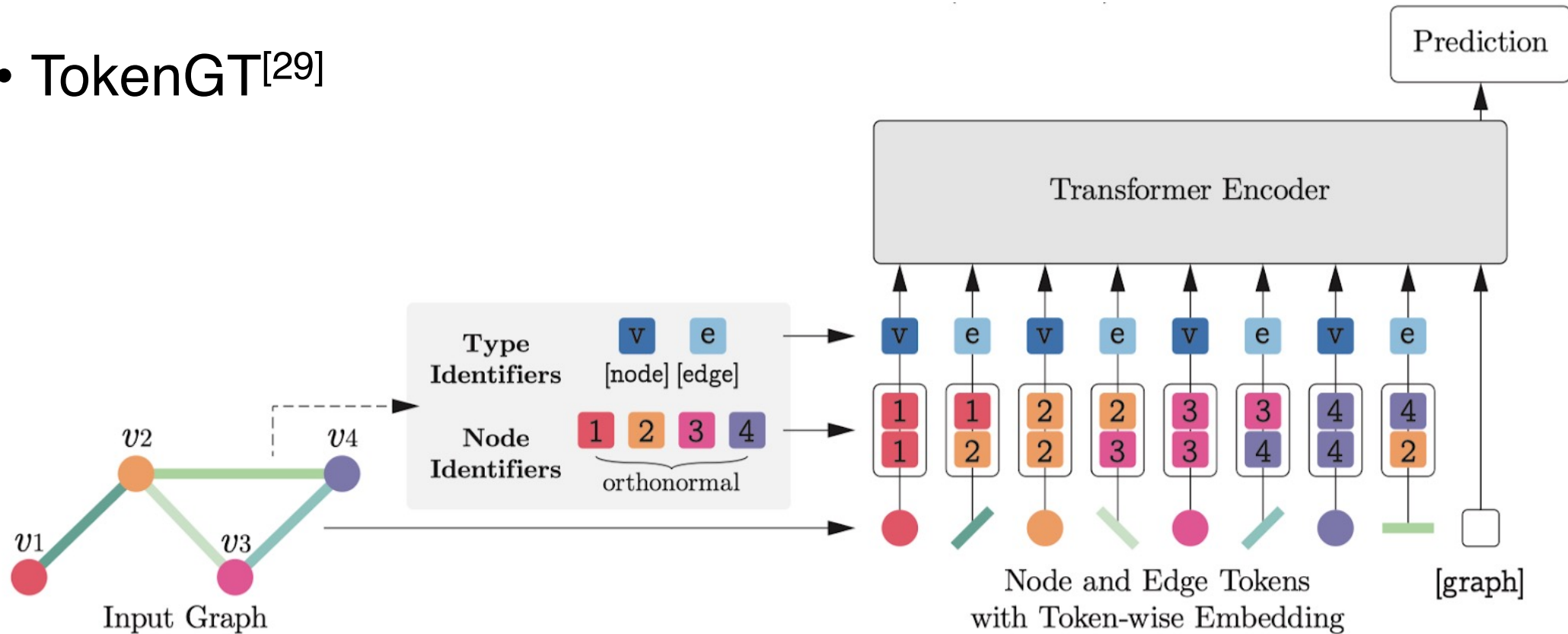
- GraphTrans^[28]



[28] Zhanghao Wu, et al. "Representing Long-Range Context for Graph Neural Networks with Global Attention"

Graph Transformer

- TokenGT^[29]



[29] Jinwoo Kim, et al. "Pure Transformers are Powerful Graph Learners"

LLM + GNN

- Large Language Model (LLM)
- "Inflection point in AI", "Dawn of Artificial General Intelligence"

Hi I'm Pi,
your personal AI.

I'm your personal AI, designed to be supportive, smart, and there for you anytime.

GitHub Copilot




 **Rowan Cheung**  
@rowancheung · [Follow](#)



I just watched GPT-4 turn a hand-drawn sketch into a functional website.

This is insane.

 **Ammaar Reshi** 
@ammaar · [Follow](#)



Can GPT-4 code an entire game for you? Yes, yes it can.

 **Pietro Schirano** 
@skirano · [Follow](#)

I don't care that it's not AGI, GPT-4 is an incredible and transformative technology.

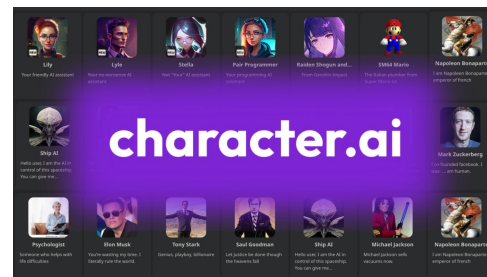
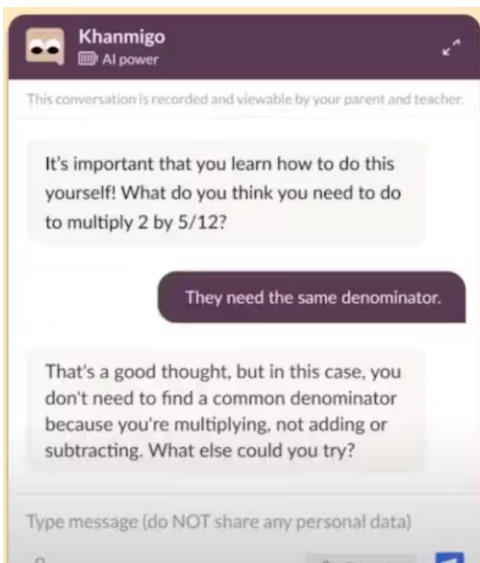
I recreated the game of Pong in under 60 seconds. It was my first try.

 **Dan Shipper**  
@danshipper · [Follow](#)



GPT-4 does drug discovery.

Give it a currently available drug and it can:



LLM + GNN

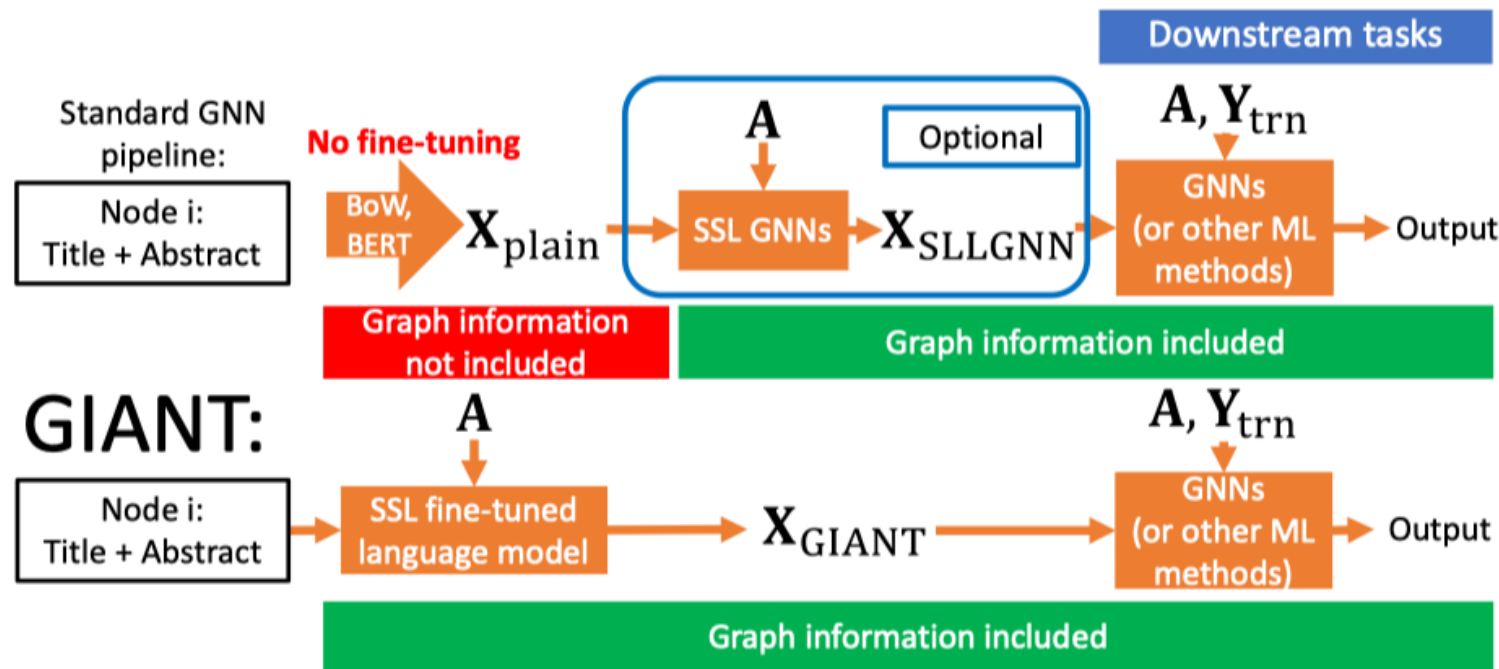
- Large Language Model (LLM)
- "Inflection point in AI", "Dawn of Artificial General Intelligence"

Can we improve GNN
using LLMs?

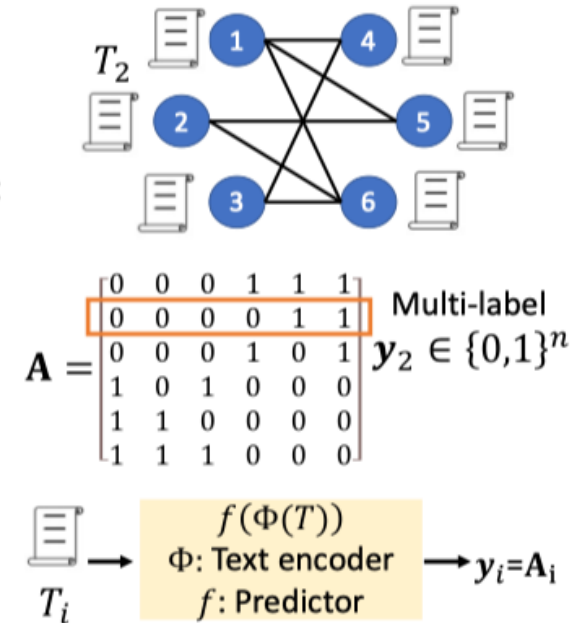


LLM + GNN

- GIANT^[30]



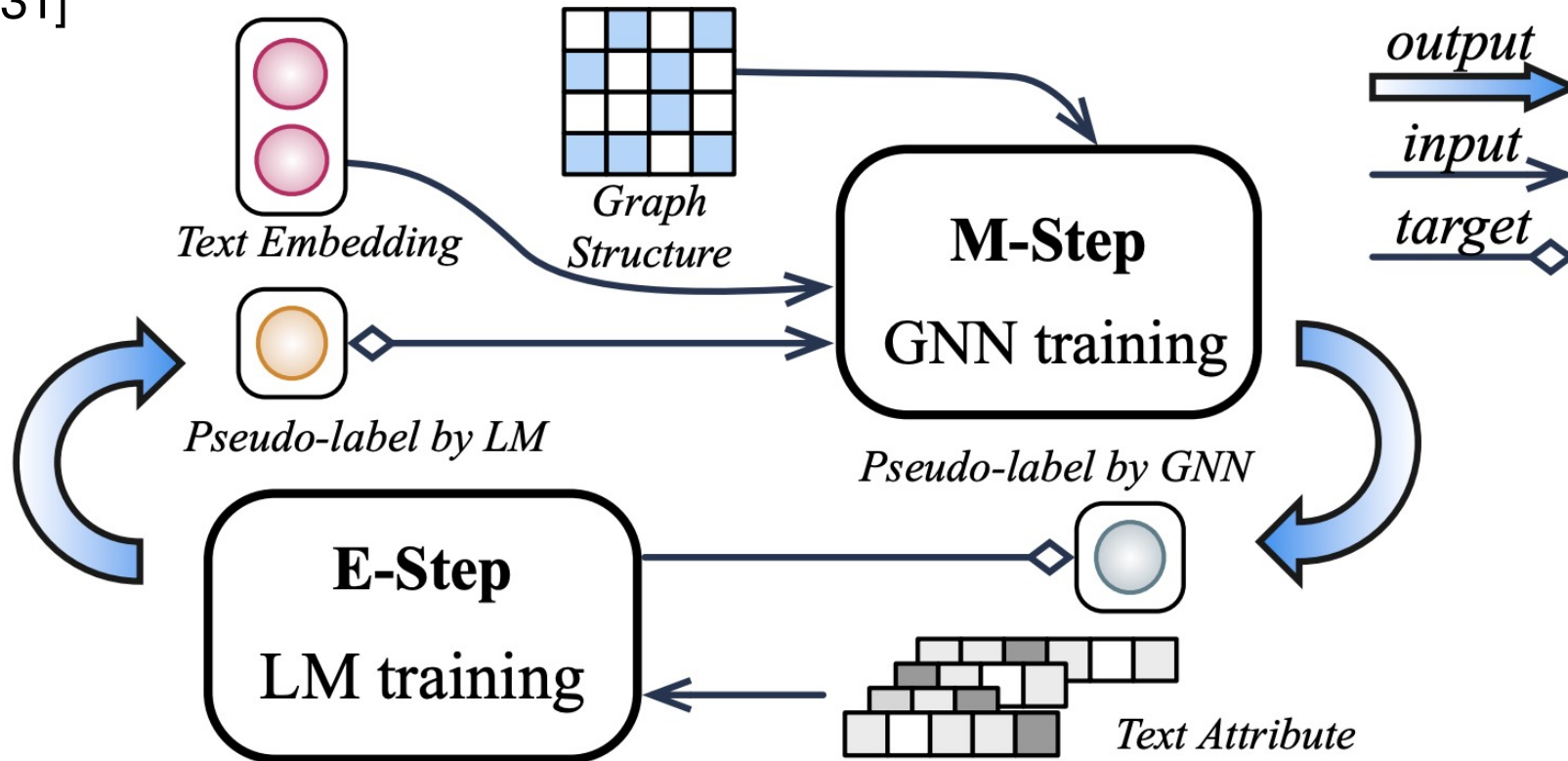
Neighborhood prediction as XMC problem:



[30] Eli Chien, et al. "Node Feature Extraction by Self-Supervised Multi-scale Neighborhood Prediction"

LLM + GNN

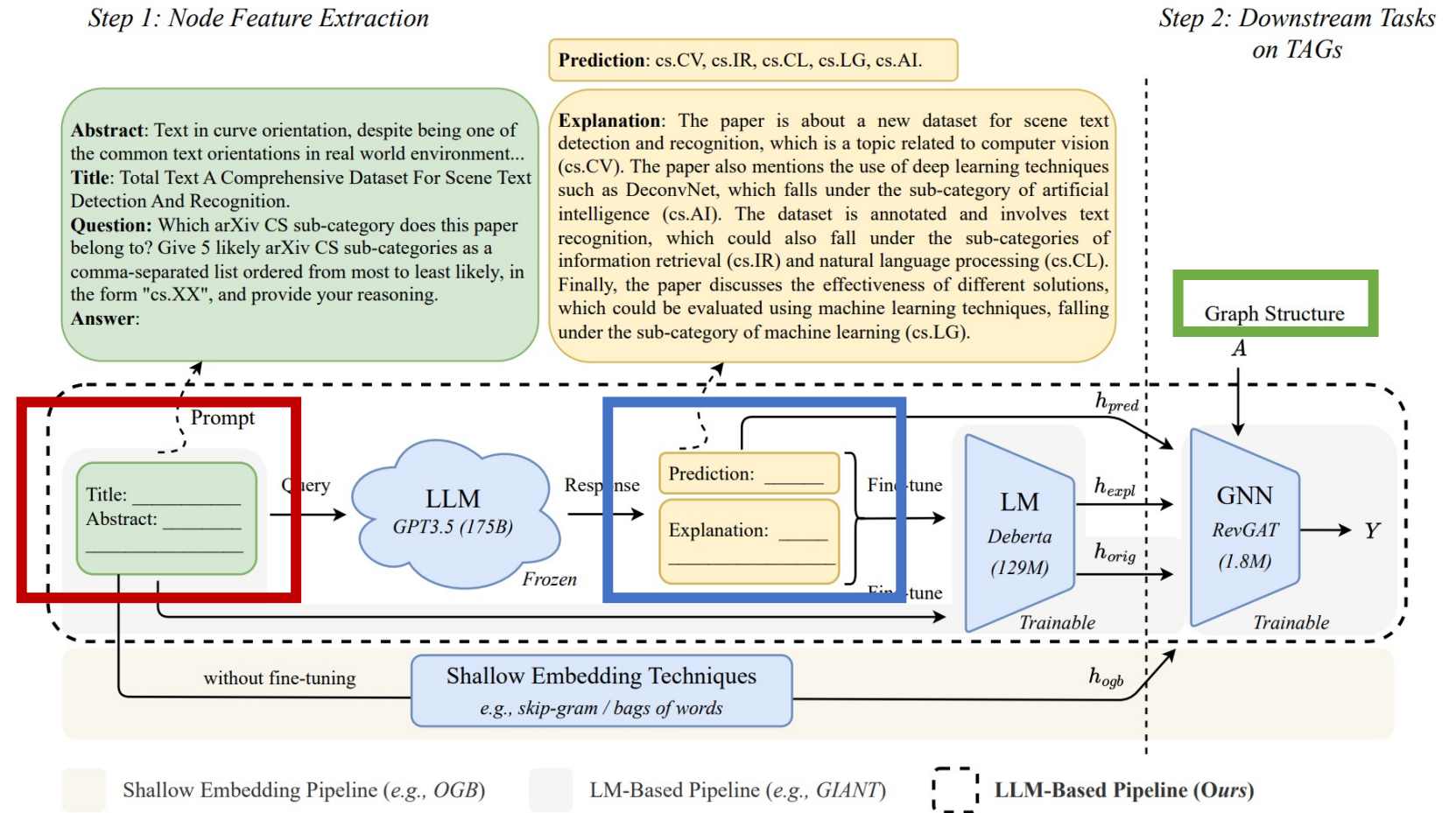
- GLEM^[31]



[31] Jianan Zhao, et al. "Learning on Large-scale Text-attributed Graphs via Variational Inference"

LLM + GNN

- TAPE^[32]



[32] Xiaoxin He, et al. "Harnessing Explanations: LLM-to-LM Interpreter for Enhanced Text-Attributed Graph Representation Learning"

Still many open problems..

- And many more chances to do groundbreaking research
- Diverse types of graphs
 - 3-dimensional graphs
 - Temporal graphs
 - Multimodal graphs
- Diverse types of architecture
 - Graph Convolution Networks
 - Graph Transformers
 - LLMs

Thank you!

Questions?

minjiy@cs.cmu.edu | <https://www.minjiyoon.xyz>